

第8章 展開 例題

1

解説

- (1) $(x+2)(y-3) = xy - 3x + 2y - 6$
- (2) $(3a+4)(2a-1) = 6a^2 - 3a + 8a - 4$
 $= 6a^2 + 5a - 4$
- (3) $(x-2y+3)(3x-y) = x(3x-y) - 2y(3x-y) + 3(3x-y)$
 $= 3x^2 - xy - 6xy + 2y^2 + 9x - 3y$
 $= 3x^2 - 7xy + 2y^2 + 9x - 3y$

2

解説

- (1) $(x+2)(x+7) = x^2 + (2+7)x + 2 \times 7$
 $= x^2 + 9x + 14$
- (2) $(x+6)(x-4) = x^2 + (6-4)x + 6 \times (-4)$
 $= x^2 + 2x - 24$
- (3) $(y-2)(y-4) = y^2 + (-2-4)y + (-2) \times (-4)$
 $= y^2 - 6y + 8$
- (4) $(a-9)(a+3) = a^2 + (-9+3)a + (-9) \times 3$
 $= a^2 - 6a - 27$
- (5) $(2x+7)(2x-9) = (2x)^2 + (7-9) \times 2x + 7 \times (-9)$
 $= 4x^2 - 4x - 63$

3

解説

- (1) $(x+6)^2 = x^2 + 2 \times 6 \times x + 6^2 = x^2 + 12x + 36$
- (2) $(3x-5)^2 = (3x)^2 - 2 \times 5 \times 3x + 5^2 = 9x^2 - 30x + 25$
- (3) $(2x+3y)^2 = (2x)^2 + 2 \times 2x \times 3y + (3y)^2 = 4x^2 + 12xy + 9y^2$

4

解説

- (1) $(x+6)(x-6) = x^2 - 6^2 = x^2 - 36$
- (2) $(2a-3b)(2a+3b) = (2a)^2 - (3b)^2 = 4a^2 - 9b^2$

5

解説

- (1) $(x-5)(x+7) = x^2 + \{(-5)+7\}x + (-5) \times 7$
 $= x^2 + 2x - 35$
- (2) $(x-8)^2 = x^2 - 2 \times 8 \times x + 8^2$
 $= x^2 - 16x + 64$
- (3) $(x+5y)(x-5y) = x^2 - (5y)^2$
 $= x^2 - 25y^2$
- (4) $(2a+3b)^2 = (2a)^2 + 2 \times 3b \times 2a + (3b)^2$
 $= 4a^2 + 12ab + 9b^2$
- (5) $(2x+y)(2x-y) = (2x)^2 - y^2$
 $= 4x^2 - y^2$
- (6) $(3x-1)(3x-4) = (3x)^2 + \{(-1)+(-4)\} \times 3x + (-1) \times (-4)$
 $= 9x^2 - 15x + 4$

6

解説

- (1) $(x-1)^2 - (x+2)(x-8) = (x^2 - 2x + 1) - (x^2 - 6x - 16)$
 $= x^2 - 2x + 1 - x^2 + 6x + 16$
 $= 4x + 17$
- (2) $3(x+2y)(x-2y) + (x-3y)^2 - (2x+3y)(2x-y)$
 $= 3(x^2 - 4y^2) + x^2 - 6xy + 9y^2 - (4x^2 - 2xy + 6xy - 3y^2)$
 $= 3x^2 - 12y^2 + x^2 - 6xy + 9y^2 - 4x^2 - 4xy + 3y^2$
 $= -10xy$

7

解説

$$(x-8)(x+2) + (4-x)(4+x) = x^2 - 6x - 16 + 16 - x^2 = -6x$$

$$x = 250 \text{ を代入すると } -6 \times 250 = -1500$$

8

解説

$$\begin{aligned} x^2 + y^2 &= (x+y)^2 - 2xy \\ &= 2^2 - 2 \times \left(-\frac{45}{4}\right) \\ &= \frac{53}{2} \quad \text{答} \end{aligned}$$

9

解説

$$\begin{aligned} (a+2b-c)^2 &= \{(a+2b)-c\}^2 \\ &= (a+2b)^2 - 2(a+2b)c + c^2 \\ &= a^2 + 4ab + 4b^2 - 2ac - 4bc + c^2 \\ &= a^2 + 4b^2 + c^2 + 4ab - 4bc - 2ca \quad \text{答} \end{aligned}$$

$$\begin{aligned} \text{別解} \quad (a+2b-c)^2 &= a^2 + (2b)^2 + (-c)^2 + 2 \times a \times 2b + 2 \times 2b \times (-c) + 2 \times (-c) \times a \\ &= a^2 + 4b^2 + c^2 + 4ab - 4bc - 2ca \quad \text{答} \end{aligned}$$

10

解説

- (1) $(x+y+3)(x+y+2) = \{(x+y)+3\}\{(x+y)+2\}$
 $= (x+y)^2 + 5(x+y) + 6$
 $= x^2 + 2xy + y^2 + 5x + 5y + 6$
- (2) $(2a-3b+1)(2a-3b-4) = \{(2a-3b)+1\}\{(2a-3b)-4\}$
 $= (2a-3b)^2 - 3(2a-3b) - 4$
 $= 4a^2 - 12ab + 9b^2 - 6a + 9b - 4$

11

解説

- (1) $(x^2+x+1)(x^2+x+3) = \{(x^2+x)+1\}\{(x^2+x)+3\}$
 $= (x^2+x)^2 + 4(x^2+x) + 3$
 $= x^4 + 2x^3 + x^2 + 4x^2 + 4x + 3$
 $= x^4 + 2x^3 + 5x^2 + 4x + 3$
- (2) $(x^2+3x+2)(x^2-3x+2) = \{(x^2+2)+3x\}\{(x^2+2)-3x\}$

$$\begin{aligned} &= (x^2+2)^2 - (3x)^2 \\ &= x^4 + 4x^2 + 4 - 9x^2 \\ &= x^4 - 5x^2 + 4 \end{aligned}$$

$$\begin{aligned} (3) \quad (a^2+ab+b^2)(a^2-ab+b^2) &= \{(a^2+b^2)+ab\}\{(a^2+b^2)-ab\} \\ &= (a^2+b^2)^2 - (ab)^2 \\ &= a^4 + 2a^2b^2 + b^4 - a^2b^2 \\ &= a^4 + a^2b^2 + b^4 \end{aligned}$$

12

解説

$$\begin{aligned} (x+1)^2(x-1)^2 &= (x+1)(x+1)(x-1)(x-1) \\ &= \{(x+1)(x-1)\}^2 \\ &= (x^2-1)^2 \\ &= (x^2)^2 - 2x^2 + 1 \\ &= x^4 - 2x^2 + 1 \quad \text{答} \end{aligned}$$

$\nearrow$ 和と差の積の公式
 $\searrow$ 差の平方の公式

13

解説

- (1) $(x-2)(x+2)(x^2+4) = \{(x-2)(x+2)\}(x^2+4)$
 $= (x^2-4)(x^2+4)$
 $= (x^2)^2 - 4^2$
 $= x^4 - 16$
- (2) $(x-5y)(x^2+25y^2)(x+5y) = (x+5y)(x-5y)(x^2+25y^2)$
 $= \{(x+5y)(x-5y)\}(x^2+25y^2)$
 $= (x^2-25y^2)(x^2+25y^2)$
 $= (x^2)^2 - (25y^2)^2$
 $= x^4 - 625y^4$
- (3) $(a+b)^2(a-b)^2(a^2+b^2)^2 = \{(a+b)(a-b)(a^2+b^2)\}^2$
 $= \{(a^2-b^2)(a^2+b^2)\}^2$
 $= (a^4-b^4)^2 = a^8 - 2a^4b^4 + b^8$

14

解説

$$\begin{aligned} &(x-1)(x+2)(x+4)(x+7) \\ &= \{(x-1)(x+7)\}\{(x+2)(x+4)\} = \{(x^2+6x)-7\}\{(x^2+6x)+8\} \\ &= (x^2+6x)^2 + (x^2+6x) - 56 = x^4 + 12x^3 + 36x^2 + x^2 + 6x - 56 \\ &= x^4 + 12x^3 + 37x^2 + 6x - 56 \end{aligned}$$

1

解説

- (1) $(x+2)(y+3) = xy+3x+2y+6$
- (2) $(x+7)(y-2) = xy-2x+7y-14$
- (3) $(a-4)(b+8) = ab+8a-4b-32$
- (4) $(k-11)(\ell-7) = k\ell-7k-11\ell+77$
- (5) $(a-2b)(c+3d) = ac+3ad-2bc-6bd$
- (6) $(p+5q)(r-13y) = pr-13py+5qr-65qy$
- (7) $(2x+1)(x+3) = 2x^2+6x+x+3 = 2x^2+7x+3$
- (8) $(2a-3)(a+5) = 2a^2+10a-3a-15 = 2a^2+7a-15$
- (9) $(8n+3)(7n-5) = 56n^2-40n+21n-15 = 56n^2-19n-15$
- (10) $(4x-y)(5x+2y) = 20x^2+8xy-5xy-2y^2 = 20x^2+3xy-2y^2$
- (11) $(9n+2m)(4m-3n) = 36mn-27n^2+8m^2-6mn = 8m^2+30mn-27n^2$
- (12) $(a+b+c)(a-b) = a(a-b)+b(a-b)+c(a-b)$
 $= a^2-ab+ab-b^2+ac-bc$
 $= a^2-b^2+ac-bc$
- (13) $(x+y+1)(x-2y+3) = x(x-2y+3)+y(x-2y+3)+(x-2y+3)$
 $= x^2-2xy+3x+xy-2y^2+3y+x-2y+3$
 $= x^2-xy+4x-2y^2+y+3$
- (14) $(x-y)(2-y-x) = x(2-y-x)-y(2-y-x)$
 $= 2x-xy-x^2-2y+y^2+xy$
 $= -x^2+2x+y^2-2y$

2

解説

- (1) $(x+3)(x+9) = x^2+(3+9)x+3 \times 9 = x^2+12x+27$
- (2) $(x-4)(x+8) = x^2+[(-4)+8]x+(-4) \times 8 = x^2+4x-32$
- (3) $(t-6)(t-7) = t^2+[(-6)+(-7)]t+(-6) \times (-7) = t^2-13t+42$
- (4) $(a+12)(a-2) = a^2+[12+(-2)]a+12 \times (-2) = a^2+10a-24$
- (5) $\left(x+\frac{1}{2}\right)\left(x+\frac{3}{2}\right) = x^2+\left(\frac{1}{2}+\frac{3}{2}\right)x+\frac{1}{2} \times \frac{3}{2} = x^2+2x+\frac{3}{4}$
- (6) $\left(x+\frac{1}{4}\right)\left(x-\frac{3}{4}\right) = x^2+\left[\frac{1}{4}+\left(-\frac{3}{4}\right)\right]x+\frac{1}{4} \times \left(-\frac{3}{4}\right) = x^2-\frac{1}{2}x-\frac{3}{16}$
- (7) $(-3+x)(x+6) = (x-3)(x+6) = x^2+[(-3)+6]x+(-3) \times 6 = x^2+3x-18$
- (8) $(2x+3)(2x+1) = (2x)^2+(3+1) \times 2x+3 \times 1 = 4x^2+8x+3$
- (9) $(4a+3)(4a-7) = (4a)^2+(3-7) \times 4a+3 \times (-7) = 16a^2-16a-21$
- (10) $(3m-1)(3m+4) = (3m)^2+(-1+4) \times 3m+(-1) \times 4 = 9m^2+9m-4$
- (11) $(8y-1)(8y-5) = (8y)^2+(-1-5) \times 8y+(-1) \times (-5) = 64y^2-48y+5$
- (12) $(5x+2)(5x-6) = (5x)^2+(2-6) \times 5x+2 \times (-6) = 25x^2-20x-12$
- (13) $(7x-3)(7x+9) = (7x)^2+(-3+9) \times 7x+(-3) \times 9 = 49x^2+42x-27$

3

解説

- (1) $(x+5)^2 = x^2+2 \times 5 \times x+5^2 = x^2+10x+25$
- (2) $(x-6)^2 = x^2-2 \times 6 \times x+6^2 = x^2-12x+36$
- (3) $(a+10)^2 = a^2+2 \times 10 \times a+10^2 = a^2+20a+100$
- (4) $(x-3)^2 = x^2-2 \times 3 \times x+3^2 = x^2-6x+9$
- (5) $(x-7)^2 = x^2-2 \times 7 \times x+7^2 = x^2-14x+49$

- (6) $(y-12)^2 = y^2-2 \times 12 \times y+12^2 = y^2-24y+144$
- (7) $(2x+5)^2 = (2x)^2+2 \times 5 \times 2x+5^2 = 4x^2+20x+25$
- (8) $(3a+2)^2 = (3a)^2+2 \times 2 \times 3a+2^2 = 9a^2+12a+4$
- (9) $(5p-8)^2 = (5p)^2-2 \times 8 \times 5p+8^2 = 25p^2-80p+64$
- (10) $(3x+2y)^2 = (3x)^2+2 \times 2y \times 3x+(2y)^2 = 9x^2+12xy+4y^2$
- (11) $(-a+4b)^2 = (-a)^2+2 \times 4b \times (-a)+(4b)^2 = a^2-8ab+16b^2$
- (12) $(7x-2y)^2 = (7x)^2-2 \times 2y \times 7x+(2y)^2 = 49x^2-28xy+4y^2$
- (13) $\left(x+\frac{1}{3}\right)^2 = x^2+2 \times \frac{1}{3} \times x+\left(\frac{1}{3}\right)^2 = x^2+\frac{2}{3}x+\frac{1}{9}$
- (14) $\left(x-\frac{1}{2}\right)^2 = x^2-2 \times \frac{1}{2} \times x+\left(\frac{1}{2}\right)^2 = x^2-x+\frac{1}{4}$
- (15) $\left(x-\frac{1}{4}\right)^2 = x^2-2 \times \frac{1}{4} \times x+\left(\frac{1}{4}\right)^2 = x^2-\frac{1}{2}x+\frac{1}{16}$

4

解説

- (1) $(x+2)(x-2) = x^2-2^2 = x^2-4$
- (2) $(a+9)(a-9) = a^2-9^2 = a^2-81$
- (3) $(x-6)(x+6) = x^2-6^2 = x^2-36$
- (4) $(x-12)(x+12) = x^2-12^2 = x^2-144$
- (5) $(5x+3y)(5x-3y) = (5x)^2-(3y)^2 = 25x^2-9y^2$
- (6) $(4a-9b)(4a+9b) = (4a)^2-(9b)^2 = 16a^2-81b^2$
- (7) $\left(x+\frac{1}{3}\right)\left(x-\frac{1}{3}\right) = x^2-\left(\frac{1}{3}\right)^2 = x^2-\frac{1}{9}$
- (8) $\left(\frac{1}{4}a-\frac{1}{5}b\right)\left(\frac{1}{4}a+\frac{1}{5}b\right) = \left(\frac{1}{4}a\right)^2-\left(\frac{1}{5}b\right)^2 = \frac{1}{16}a^2-\frac{1}{25}b^2$
- (9) $\left(\frac{1}{7}b+a\right)\left(a-\frac{1}{7}b\right) = \left(a+\frac{1}{7}b\right)\left(a-\frac{1}{7}b\right) = a^2-\left(\frac{1}{7}b\right)^2 = a^2-\frac{1}{49}b^2$

5

解説

- (1) $(x-4)(x-5) = x^2-9x+20$
- (2) $(x+6)^2 = x^2+12x+36$
- (3) $(a+7)(a-7) = a^2-49$
- (4) $(x-2)(x+5) = x^2+3x-10$
- (5) $(x-2)^2 = x^2-4x+4$
- (6) $(3x+4)(3x-4) = 9x^2-16$
- (7) $(a-4b)^2 = a^2-8ab+16b^2$
- (8) $(4a-5b)(4a+5b) = 16a^2-25b^2$
- (9) $(x+2)(x+7) = x^2+9x+14$
- (10) $(2x+5y)^2 = 4x^2+20xy+25y^2$
- (11) $(3x+5)(3x-5) = 9x^2-25$
- (12) $(5x-2y)^2 = 25x^2-20xy+4y^2$
- (13) $(2a+5b)(2a-5b) = 4a^2-25b^2$
- (14) $(x+6)(x-4) = x^2+2x-24$
- (15) $(x+4)(x-4) = x^2-16$
- (16) $(a+3)(a-8) = a^2-5a-24$
- (17) $(2a-3)^2 = 4a^2-12a+9$
- (18) $(3x-4y)^2 = 9x^2-24xy+16y^2$
- (19) $(2x-3)(2x+3) = 4x^2-9$
- (20) $(x+5)(x-5) = x^2-25$
- (21) $(y-2)(y-4) = y^2-6y+8$
- (22) $(x-4)(x+2) = x^2-2x-8$
- (23) $(3x+5y)^2 = 9x^2+30xy+25y^2$
- (24) $(2a+b)(2a-b) = 4a^2-b^2$
- (25) $(p-7)(p+6) = p^2-p-42$
- (26) $(a-4b)(a-7b) = a^2-11ab+28b^2$
- (27) $(2a+3x)(2a-3x) = 4a^2-9x^2$
- (28) $(5x-2y)^2 = 25x^2-20xy+4y^2$
- (29) $(x-5y)(x+2y) = x^2-3xy-10y^2$
- (30) $(3a-2b)^2 = 9a^2-12ab+4b^2$

6

解説

- (1) $(x+6)^2-(x+12)(x+3) = (x^2+12x+36)-(x^2+15x+36)$
 $= x^2+12x+36-x^2-15x-36$
 $= -3x$
- (2) $(x-3)(x+15)-(x-6)^2 = (x^2+12x-45)-(x^2-12x+36)$
 $= x^2+12x-45-x^2+12x-36$
 $= 24x-81$
- (3) $(2x+3)^2-(2x+5)(2x-5) = (4x^2+12x+9)-(4x^2-25)$
 $= 4x^2+12x+9-4x^2+25$
 $= 12x+34$
- (4) $(2a-3)(5a+2)+(a+5)(a+6) = (10a^2-11a-6)+(a^2+11a+30)$
 $= 10a^2-11a-6+a^2+11a+30$
 $= 11a^2+24$
- (5) $(2b-a)^2-(a+5b)(a-b) = (4b^2-4ab+a^2)-(a^2+4ab-5b^2)$
 $= 4b^2-4ab+a^2-a^2-4ab+5b^2$
 $= 9b^2-8ab$
- (6) $(6x+5y)(3x-4y)+(x+4y)(2x+y) = (18x^2-9xy-20y^2)+(2x^2+9xy+4y^2)$
 $= 18x^2-9xy-20y^2+2x^2+9xy+4y^2$
 $= 20x^2-16y^2$
- (7) $(3x-2y)(2y+3x)+(5x-2y)^2 = (3x-2y)(3x+2y)+(5x-2y)^2$
 $= (9x^2-4y^2)+(25x^2-20xy+4y^2)$
 $= 9x^2-4y^2+25x^2-20xy+4y^2$
 $= 34x^2-20xy$
- (8) $(2a-6b)^2+(4a+3b)^2 = (4a^2-24ab+36b^2)+(16a^2+24ab+9b^2)$
 $= 4a^2-24ab+36b^2+16a^2+24ab+9b^2$
 $= 20a^2+45b^2$
- (9) $3(a+4)(a-5)-2(a-1)(a+2) = 3(a^2-a-20)-2(a^2+a-2)$
 $= 3a^2-3a-60-2a^2-2a+4$
 $= a^2-5a-56$

7

解説

$$(2a+3)^2-4a(a+5) = 4a^2+12a+9-4a^2-20a = -8a+9$$

この式に $a = -\frac{1}{8}$ を代入すると、求める式の値は

$$-8 \times \left(-\frac{1}{8}\right) + 9 = 1 + 9 = 10$$

8

解説

$$x^2+y^2 = (x+y)^2-2xy$$

$$= \left(\frac{17}{2}\right)^2 - 2 \times 18 = \frac{289}{4} - 36 = \frac{145}{4}$$

9

解説

$$\begin{aligned}
(1) \quad & (a-b+1)^2 = [(a-b)+1]^2 \\
& = (a-b)^2 + 2(a-b) \times 1 + 1^2 \\
& = a^2 - 2ab + b^2 + 2a - 2b + 1 \\
(2) \quad & (2a-b-c)^2 = [(2a-b)-c]^2 \\
& = (2a-b)^2 - 2(2a-b)c + c^2 \\
& = 4a^2 - 4ab + b^2 - 4ac + 2bc + c^2 \\
& = 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac \\
(3) \quad & (a+2b+3c)^2 = [(a+2b)+3c]^2 \\
& = (a+2b)^2 + 2(a+2b) \times 3c + (3c)^2 \\
& = a^2 + 4ab + 4b^2 + 6ac + 12bc + 9c^2 \\
& = a^2 + 4b^2 + 9c^2 + 4ab + 12bc + 6ca \\
(4) \quad & (3x+5y+4z)^2 = [(3x+5y)+4z]^2 \\
& = (3x+5y)^2 + 2(3x+5y) \times 4z + (4z)^2 \\
& = 9x^2 + 30xy + 25y^2 + 24xz + 40yz + 16z^2 \\
& = 9x^2 + 25y^2 + 16z^2 + 30xy + 40yz + 24xz \\
(5) \quad & (9x+8y-6z)^2 = [(9x+8y)-6z]^2 \\
& = (9x+8y)^2 - 2(9x+8y) \times 6z + (6z)^2 \\
& = 81x^2 + 144xy + 64y^2 - 108xz - 96yz + 36z^2 \\
& = 81x^2 + 64y^2 + 36z^2 + 144xy - 96yz - 108xz \\
(6) \quad & (-7x+5y-3z)^2 = [(-7x+5y)-3z]^2 \\
& = (-7x+5y)^2 - 2(-7x+5y) \times 3z + (3z)^2 \\
& = 49x^2 - 70xy + 25y^2 + 42xz - 30yz + 9z^2 \\
& = 49x^2 + 25y^2 + 9z^2 - 70xy - 30yz + 42xz
\end{aligned}$$

例解 公式 $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ を使って展開すると、(1)、(2)は次のようになる。

$$\begin{aligned}
(1) \quad & (a-b+1)^2 = a^2 + (-b)^2 + 1^2 + 2 \times a \times (-b) + 2 \times (-b) \times 1 + 2 \times 1 \times a \\
& = a^2 - 2ab + b^2 + 2a - 2b + 1 \\
(2) \quad & (2a-b-c)^2 = (2a)^2 + (-b)^2 + (-c)^2 + 2 \times 2a \times (-b) + 2 \times (-b) \times (-c) \\
& \quad \quad \quad + 2 \times (-c) \times 2a \\
& = 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ca
\end{aligned}$$

(3)~(6)も同様に公式を使って展開できる。

10

解説

$$\begin{aligned}
(1) \quad & (x+y+8)(x+y+7) = [(x+y)+8][(x+y)+7] \\
& = (x+y)^2 + 15(x+y) + 56 \\
& = x^2 + 2xy + y^2 + 15x + 15y + 56 \\
(2) \quad & (a+b+3)(a+b-4) = [(a+b)+3][(a+b)-4] \\
& = (a+b)^2 - (a+b) - 12 \\
& = a^2 + 2ab + b^2 - a - b - 12 \\
(3) \quad & (x-2y+3)(x-2y-9) = [(x-2y)+3][(x-2y)-9] \\
& = (x-2y)^2 - 6(x-2y) - 27 \\
& = x^2 - 4xy + 4y^2 - 6x + 12y - 27 \\
(4) \quad & (3a-2b+5)(3a-2b-3) = [(3a-2b)+5][(3a-2b)-3]
\end{aligned}$$

$$\begin{aligned}
& = (3a-2b)^2 + 2(3a-2b) - 15 \\
& = 9a^2 - 12ab + 4b^2 + 6a - 4b - 15 \\
(5) \quad & (2x+3y-z)(2x+3y+z) = [(2x+3y)-z][(2x+3y)+z] \\
& = (2x+3y)^2 - z^2 \\
& = 4x^2 + 12xy + 9y^2 - z^2 \\
(6) \quad & (4p-5q+2r)(4p-5q-3r) = [(4p-5q)+2r][(4p-5q)-3r] \\
& = (4p-5q)^2 - r(4p-5q) - 6r^2 \\
& = 16p^2 - 40pq + 25q^2 - 4pr + 5qr - 6r^2 \\
& = 16p^2 + 25q^2 - 6r^2 - 40pq + 5qr - 4rp
\end{aligned}$$

11

解説

$$\begin{aligned}
(1) \quad & (a^2+3a+4)(a^2+3a-7) = [(a^2+3a)+4][(a^2+3a)-7] \\
& = (a^2+3a)^2 - 3(a^2+3a) - 28 \\
& = a^4 + 6a^3 + 9a^2 - 3a^2 - 9a - 28 \\
& = a^4 + 6a^3 + 6a^2 - 9a - 28 \\
(2) \quad & (x^2+6x+1)(x^2+6x-1) = [(x^2+6x)+1][(x^2+6x)-1] \\
& = (x^2+6x)^2 - 1^2 \\
& = x^4 + 12x^3 + 36x^2 - 1 \\
(3) \quad & (x^2-3xy+y^2)(x^2+3xy+y^2) = [(x^2+y^2)-3xy][(x^2+y^2)+3xy] \\
& = (x^2+y^2)^2 - (3xy)^2 \\
& = x^4 + 2x^2y^2 + y^4 - 9x^2y^2 \\
& = x^4 - 7x^2y^2 + y^4 \\
(4) \quad & (a^2+3ab-2b^2)(a^2-3ab-2b^2) = [(a^2-2b^2)+3ab][(a^2-2b^2)-3ab] \\
& = (a^2-2b^2)^2 - (3ab)^2 \\
& = a^4 - 4a^2b^2 + 4b^4 - 9a^2b^2 \\
& = a^4 - 13a^2b^2 + 4b^4
\end{aligned}$$

12

解説

$$\begin{aligned}
(1) \quad & (a-5)^2(a+5)^2 = [(a-5)(a+5)]^2 \\
& = (a^2-25)^2 = a^4 - 50a^2 + 625 \\
(2) \quad & (x+4)^2(x-4)^2 = [(x+4)(x-4)]^2 \\
& = (x^2-16)^2 = x^4 - 32x^2 + 256 \\
(3) \quad & (p+3q)^2(p-3q)^2 = [(p+3q)(p-3q)]^2 \\
& = (p^2-9q^2)^2 = p^4 - 18p^2q^2 + 81q^4 \\
(4) \quad & (2x-y)^2(2x+y)^2 = [(2x-y)(2x+y)]^2 \\
& = (4x^2-y^2)^2 = 16x^4 - 8x^2y^2 + y^4 \\
(5) \quad & (3a-4b)^2(3a+4b)^2 = [(3a-4b)(3a+4b)]^2 = (9a^2-16b^2)^2 \\
& = 81a^4 - 288a^2b^2 + 256b^4 \\
(6) \quad & (7y+2x)^2(2x-7y)^2 = [(2x+7y)(2x-7y)]^2 = (4x^2-49y^2)^2 \\
& = 16x^4 - 392x^2y^2 + 2401y^4
\end{aligned}$$

13

解説

$$\begin{aligned}
(1) \quad & (x-4)(x+4)(x^2+16) = [(x-4)(x+4)](x^2+16) \\
& = (x^2-16)(x^2+16)
\end{aligned}$$

$$\begin{aligned}
& = (x^2)^2 - 16^2 \\
& = x^4 - 256 \\
(2) \quad & (3x-y)(9x^2+y^2)(3x+y) = (3x+y)(3x-y)(9x^2+y^2) \\
& = [(3x+y)(3x-y)](9x^2+y^2) \\
& = (9x^2-y^2)(9x^2+y^2) \\
& = (9x^2)^2 - (y^2)^2 \\
& = 81x^4 - y^4
\end{aligned}$$

$$\begin{aligned}
(3) \quad & (x-y)^2(x+y)^2(x^2+y^2)^2 = [(x-y)(x+y)]^2(x^2+y^2)^2 \\
& = (x^2-y^2)^2(x^2+y^2)^2 \\
& = [(x^2-y^2)(x^2+y^2)]^2 \\
& = (x^4-y^4)^2 \\
& = x^8 - 2x^4y^4 + y^8
\end{aligned}$$

$$\begin{aligned}
(4) \quad & (2a-3b)^2(2a+3b)^2(4a^2+9b^2)^2 = [(2a-3b)(2a+3b)]^2(4a^2+9b^2)^2 \\
& = (4a^2-9b^2)^2(4a^2+9b^2)^2 \\
& = [(4a^2-9b^2)(4a^2+9b^2)]^2 \\
& = [(4a^2)^2 - (9b^2)^2]^2 \\
& = (16a^4 - 81b^4)^2 \\
& = 256a^8 - 2592a^4b^4 + 6561b^8
\end{aligned}$$

14

解説

$$\begin{aligned}
(1) \quad & (x-1)(x-2)(x-3)(x-4) = (x-1)(x-4) \times (x-2)(x-3) \\
& = (x^2-5x+4)(x^2-5x+6) \\
& = [(x^2-5x)+4][(x^2-5x)+6] \\
& = (x^2-5x)^2 + 10(x^2-5x) + 24 \\
& = x^4 - 10x^3 + 25x^2 + 10x^2 - 50x + 24 \\
& = x^4 - 10x^3 + 35x^2 - 50x + 24 \\
(2) \quad & (x-1)(x-2)(x+3)(x+4) = [(x-1)(x+3)][(x-2)(x+4)] \\
& = [(x^2+2x)-3][(x^2+2x)-8] \\
& = (x^2+2x)^2 - 11(x^2+2x) + 24 \\
& = x^4 + 4x^3 + 4x^2 - 11x^2 - 22x + 24 \\
& = x^4 + 4x^3 - 7x^2 - 22x + 24 \\
(3) \quad & (x-2)(x-3)(x+4)(x+5) = (x-2)(x+4) \times (x-3)(x+5) \\
& = (x^2+2x-8)(x^2+2x-15) \\
& = [(x^2+2x)-8][(x^2+2x)-15] \\
& = (x^2+2x)^2 - 23(x^2+2x) + 120 \\
& = x^4 + 4x^3 + 4x^2 - 23x^2 - 46x + 120 \\
& = x^4 + 4x^3 - 19x^2 - 46x + 120 \\
(4) \quad & (x+1)(x+3)(x-5)(x-7) = [(x+1)(x-5)][(x+3)(x-7)] \\
& = (x^2-4x-5)(x^2-4x-21) \\
& = [(x^2-4x)-5][(x^2-4x)-21] \\
& = (x^2-4x)^2 - 26(x^2-4x) + 105 \\
& = x^4 - 8x^3 + 16x^2 - 26x^2 + 104x + 105 \\
& = x^4 - 8x^3 - 10x^2 + 104x + 105
\end{aligned}$$

1

解説

$$(1) \quad (3a+2)^2=(3a)^2+2\cdot 3a\cdot 2+2^2 \\ =9a^2+12a+4$$

$$(2) \quad (5x-2y)^2=(5x)^2-2\cdot 5x\cdot 2y+(2y)^2 \\ =25x^2-20xy+4y^2$$

$$(3) \quad (4x+3)(4x-3)=(4x)^2-3^2 \\ =16x^2-9$$

$$(4) \quad (-2b-a)(a-2b)=-(a+2b)(a-2b)=-[a^2-(2b)^2] \\ =-(a^2-4b^2) \\ =-a^2+4b^2$$

$$(5) \quad (x+6)(x+7)=x^2+(6+7)x+6\cdot 7 \\ =x^2+13x+42$$

$$(6) \quad (2t-3)(2t-5)=(2t)^2+((-3)+(-5))\cdot 2t+(-3)(-5) \\ =4t^2-16t+15$$

$$(7) \quad (4x+1)(3x-2)=4\cdot 3x^2+(4\cdot (-2)+1\cdot 3)x+1\cdot (-2) \\ =12x^2-5x-2$$

$$(8) \quad (2a+3b)(3a+5b)=2\cdot 3a^2+[2\cdot (5b)+(3b)\cdot 3]a+3b\cdot 5b \\ =6a^2+19ab+15b^2$$

$$(9) \quad (7x-3)(-2x+3)=7\cdot (-2)x^2+[7\cdot 3+(-3)\cdot (-2)]x+(-3)\cdot 3 \\ =-14x^2+27x-9$$

2

解説

$$(1) \quad \text{与式}=(3x)^2+2\cdot 3x\cdot 5+5^2=9x^2+30x+25$$

$$(2) \quad \text{与式}=(2p)^2-2\cdot 2p\cdot 5+5^2=4p^2-20p+25$$

$$(3) \quad \text{与式}=(4a)^2-2\cdot 4a\cdot 3b+(3b)^2=16a^2-24ab+9b^2$$

$$(4) \quad \text{与式}=(-x^2)^2-2\cdot (-x^2)\cdot x+x^2=x^4+2x^3+x^2$$

$$(5) \quad \text{与式}=(3a)^2-2\cdot 3a\cdot \frac{1}{2}b+\left(\frac{1}{2}b\right)^2=9a^2-3ab+\frac{1}{4}b^2$$

$$(6) \quad \text{与式}=(3x)^2-5^2=9x^2-25$$

$$(7) \quad \text{与式}=(2a)^2-(7b)^2=4a^2-49b^2$$

$$(8) \quad \text{与式}=(a^2)^2-(b^2)^2=a^4-b^4$$

$$(9) \quad \text{与式}=(-ab)^2-c^2=a^2b^2-c^2$$

3

解説

$$(1) \quad (x+2)(x+\text{○})=x^2+6x+\text{○} \\ 2+\text{○}=6, \quad 2\times\text{○}=\text{○} \\ \text{よって} \quad \text{○}=4, \quad \text{○}=8$$

$$(2) \quad (x-\text{○})(x+5)=x^2-\text{○}x-35 \\ -\text{○}+5=-\text{○}, \quad -(\text{○})\times 5=-35 \\ \text{よって} \quad \text{○}=7, \quad \text{○}=2$$

$$(3) \quad (x+\text{○})^2=x^2+12x+\text{○} \\ 2\times\text{○}=12, \quad \text{○}^2=\text{○} \\ \text{よって} \quad \text{○}=6, \quad \text{○}=36$$

$$(4) \quad (\text{○}x-3)^2=\text{○}x^2-30x+9$$

$$\text{○}^2=\text{○}, \quad 2\times\text{○}\times 3=30$$

$$\text{よって} \quad \text{○}=5, \quad \text{○}=25$$

$$(5) \quad (3x+\text{○})(3x-8)=9x^2-\text{○}x$$

$$\text{○}=8, \quad \text{○}=64$$

$$(6) \quad (7x-2)(\text{○}x+2)=\text{○}x^2-4x$$

$$\text{○}=7, \quad \text{○}=49$$

4

解説

$$(1) \quad (ab-6)(ab+3)=a^2b^2-3ab-18 \quad (2) \quad \left(x-\frac{1}{2}y\right)\left(x+\frac{1}{2}y\right)=x^2-\frac{1}{4}y^2$$

$$(3) \quad (-2a+5b)^2=4a^2-20ab+25b^2 \quad (4) \quad (2a+3)(2a+7)=4a^2+20a+21$$

$$(5) \quad (-ab+c)(-ab-c)=a^2b^2-c^2 \quad (6) \quad (7y-2)(7y-6)=49y^2-56y+12$$

$$(7) \quad (a^2+2b)^2=a^4+4a^2b+4b^2 \quad (8) \quad (4x-y)(y+4x)=16x^2-y^2$$

$$(9) \quad (a^2-3)(a^2+7)=a^4+4a^2-21 \quad (10) \quad (a^2+b^2)(a^2-b^2)=a^4-b^4$$

$$(11) \quad (2t-3)(2t-5)=4t^2-16t+15 \quad (12) \quad (4x-3)(4x+9)=16x^2+24x-27$$

$$(13) \quad (2xy-3)^2=4x^2y^2-12xy+9 \quad (14) \quad \left(\frac{1}{4}a-\frac{1}{5}b\right)\left(\frac{1}{4}a+\frac{1}{5}b\right)=\frac{1}{16}a^2-\frac{1}{25}b^2$$

$$(15) \quad \left(\frac{1}{7}b+a\right)\left(a-\frac{1}{7}b\right)=a^2-\frac{1}{49}b^2 \quad (16) \quad (3a+1)(3a+5)=9a^2+18a+5$$

$$(17) \quad (4x-1)(4x+5)=16x^2+16x-5 \quad (18) \quad (x^2+x)^2=x^4+2x^3+x^2$$

$$(19) \quad (-x^2-x)^2=x^4+2x^3+x^2 \quad (20) \quad (ab+3)(ab-4)=a^2b^2-ab-12$$

$$(21) \quad (3a-4b)(4b+3a)=9a^2-16b^2 \quad (22) \quad (xy+ab)(xy+7ab)=x^2y^2+8abxy+7a^2b^2$$

$$(23) \quad \left(a+\frac{1}{2}b\right)^2=a^2+ab+\frac{1}{4}b^2 \quad (24) \quad (4a+3b)(4a-7b)=16a^2-16ab-21b^2$$

$$(25) \quad \left(x-\frac{3}{5}\right)\left(x-\frac{5}{6}\right)=x^2-\frac{43}{30}x+\frac{1}{2} \quad (26) \quad (5x+6)(5x-8)=25x^2-10x-48$$

$$(27) \quad (x^3+4x)^2=x^6+8x^4+16x^2 \quad (28) \quad (8y-1)(8y-5)=64y^2-48y+5$$

$$(29) \quad (2x^4-5y^5)^2=4x^8-20x^4y^5+25y^{10} \quad (30) \quad (a^3+b^3)(a^3-b^3)=a^6-b^6$$

5

解説

$$(1) \quad (x+2)(x-8)+(x+3)^2=(x^2-6x-16)+(x^2+6x+9)$$

$$=x^2-6x-16+x^2+6x+9 \\ =2x^2-7$$

$$(2) \quad (x-3)(x+5)-(x+6)(x-7)=(x^2+2x-15)-(x^2-x-42)$$

$$=x^2+2x-15-x^2+x+42 \\ =3x+27$$

$$(3) \quad (3x-2)^2-(x+1)(x-4)=(9x^2-12x+4)-(x^2-3x-4)$$

$$=9x^2-12x+4-x^2+3x+4 \\ =8x^2-9x+8$$

$$(4) \quad (2m+3n)^2+(2m-3n)^2=(4m^2+12mn+9n^2)+(4m^2-12mn+9n^2)$$

$$=4m^2+12mn+9n^2+4m^2-12mn+9n^2 \\ =8m^2+18n^2$$

$$(5) \quad (x+2y)(x-y)-(3x-y)^2=(x^2+xy-2y^2)-(9x^2-6xy+y^2)$$

$$=x^2+xy-2y^2-9x^2+6xy-y^2 \\ =-8x^2+7xy-3y^2$$

$$(6) \quad (-a+3)(-a-3)+2(a+1)^2=(a^2-9)+2(a^2+2a+1)$$

$$=a^2-9+2a^2+4a+2$$

$$=3a^2+4a-7$$

$$(7) \quad (x+3)(x-7)+(x-3)^2-(2x-3)(x-5)$$

$$=(x^2-4x-21)+(x^2-6x+9)-(2x^2-13x+15) \\ =x^2-4x-21+x^2-6x+9-2x^2+13x-15 \\ =3x-27$$

$$(8) \quad (5a+3b)(5a-3b)-(2a-7b)^2-(a-4b)(a+2b)$$

$$=(25a^2-9b^2)-(4a^2-28ab+49b^2)-(a^2-2ab-8b^2) \\ =25a^2-9b^2-4a^2+28ab-49b^2-a^2+2ab+8b^2 \\ =20a^2+30ab-50b^2$$

6

解説

$$(1) \quad (x+2y+1)(x+2y-1)=((x+2y)+1)((x+2y)-1)$$

$$=(x+2y)^2-1^2 \\ =x^2+4xy+4y^2-1$$

$$(2) \quad (a-3b+c)(a-3b-c)=((a-3b)+c)((a-3b)-c)$$

$$=(a-3b)^2-c^2 \\ =a^2-6ab+9b^2-c^2$$

$$(3) \quad (x^2+6x+2)(x^2+6x-2)=((x^2+6x)+2)((x^2+6x)-2)$$

$$=(x^2+6x)^2-2^2 \\ =x^4+12x^3+36x^2-4$$

$$(4) \quad (x-3xy+y)(x+3xy+y)=((x+y)-3xy)((x+y)+3xy)$$

$$=(x+y)^2-(3xy)^2 \\ =x^2+2xy+y^2-9x^2y^2$$

$$(5) \quad (a^2+3ab-2b^2)(a^2-3ab-2b^2)=((a^2-2b^2)+3ab)((a^2-2b^2)-3ab)$$

$$=(a^2-2b^2)^2-(3ab)^2 \\ =a^4-4a^2b^2+4b^4-9a^2b^2 \\ =a^4-13a^2b^2+4b^4$$

7

解説

$$(1) \quad (x^2-3x+1)(x^2+3x+1)=((x^2+1)-3x)((x^2+1)+3x)$$

$$=(x^2+1)^2-9x^2 \\ =x^4-7x^2+1$$

$$(2) \quad (\text{与式})=((x^2+2)+x)((x^2+2)-x)=(x^2+2)^2-x^2=x^4+3x^2+4$$

$$(3) \quad (x^2+3x+2)(x^2-3x+2)=((x^2+2)+3x)((x^2+2)-3x)=(x^2+2)^2-(3x)^2 \\ =x^4-5x^2+4$$

$$(4) \quad (\text{与式})=((a^2+9)+3a)((a^2+9)-3a)=(a^2+9)^2-(3a)^2$$

$$=a^4+18a^2+81-9a^2=a^4+9a^2+81$$

$$(5) \quad (x^2-2xy+4y^2)(x^2+2xy+4y^2)=(x^2+4y^2)^2-(2xy)^2$$

$$=x^4+8x^2y^2+16y^4-4x^2y^2 \\ =x^4+4x^2y^2+16y^4$$

8

解説

$$(1) \quad (\text{与式}) = \{(x^2 + 4x) + 1\}[(x^2 + 4x) + 3] = (x^2 + 4x)^2 + (1 + 3)(x^2 + 4x) + 1 \cdot 3 \\ = (x^2)^2 + 2 \cdot x^2 \cdot 4x + (4x)^2 + 4 \cdot x^2 + 4 \cdot 4x + 3 = x^4 + 8x^3 + 16x^2 + 4x^2 + 16x + 3 \\ = x^4 + 8x^3 + 20x^2 + 16x + 3$$

$$(2) \quad (\text{与式}) = \{(x^2 - 1) + 2x\}[(x^2 - 1) + 3x] = (x^2 - 1)^2 + (2x + 3x)(x^2 - 1) + 2x \cdot 3x \\ = (x^2)^2 - 2 \cdot x^2 \cdot 1 + 1^2 + 5x(x^2 - 1) + 6x^2 = x^4 - 2x^2 + 1 + 5x^3 - 5x + 6x^2 \\ = x^4 + 5x^3 + 4x^2 - 5x + 1$$

$$(3) \quad (\text{与式}) = \{x + (2y - 3z)\}[x - (2y - 3z)] = x^2 - (2y - 3z)^2 \\ = x^2 - \{(2y)^2 - 2 \cdot 2y \cdot 3z + (3z)^2\} = x^2 - 4y^2 + 12yz - 9z^2$$

$$(4) \quad (\text{与式}) = \{a^2 - (ab - b^2)\}[\{a^2 + (ab - b^2)\} = (a^2)^2 - (ab - b^2)^2 \\ = a^4 - \{(ab)^2 - 2 \cdot ab \cdot b^2 + (b^2)^2\} = a^4 - a^2b^2 + 2ab^3 - b^4$$

$$(5) \quad (\text{与式}) = \{x^2 + (2 + 3)x + 2 \cdot 3\}(x^2 + 5x) = \{(x^2 + 5x) + 6\}(x^2 + 5x) \\ = (x^2 + 5x)^2 + 6(x^2 + 5x) = (x^2)^2 + 2 \cdot x^2 \cdot 5x + (5x)^2 + 6x^2 + 30x \\ = x^4 + 10x^3 + 31x^2 + 30x$$

$$(6) \quad (\text{与式}) = (x^2 - 6)\{x^2 + (2 - 3)x + 2 \cdot (-3)\} = (x^2 - 6)(x^2 - x - 6) \\ = (x^2 - 6)\{(x^2 - 6) - x\} = (x^2 - 6)^2 - (x^2 - 6)x \\ = (x^2)^2 - 2 \cdot x^2 \cdot 6 + 6^2 - x^3 + 6x = x^4 - x^3 - 12x^2 + 6x + 36$$

$$(7) \quad (\text{与式}) = (x^2 - 4)(x^2 - 9) = (x^2)^2 - (4 + 9)x^2 + (-4) \cdot (-9) \\ = x^4 - 13x^2 + 36$$

$$(8) \quad (\text{与式}) = (x - 1)(x + 1) \times (x - 3)(x + 3) = (x^2 - 1)(x^2 - 9) = x^4 - 10x^2 + 9$$

$$(9) \quad (\text{与式}) = (x + 1)(x - 4) \times (x - 1)(x - 2) = (x^2 - 3x - 4)(x^2 - 3x + 2) \\ = \{(x^2 - 3x) - 4\}[\{(x^2 - 3x) + 2\}] = (x^2 - 3x)^2 - 2(x^2 - 3x) - 8 \\ = x^4 - 6x^3 + 9x^2 - 2x^2 + 6x - 8 = x^4 - 6x^3 + 7x^2 + 6x - 8$$

$$(10) \quad (\text{与式}) = (x + 1)(x + 6) \times (x + 2)(x + 3) = (x^2 + 7x + 6)(x^2 + 5x + 6) \\ = \{(x^2 + 6) + 7x\}[(x^2 + 6) + 5x] = (x^2 + 6)^2 + 12x(x^2 + 6) + 35x^2 \\ = x^4 + 12x^2 + 36 + 12x^3 + 72x + 35x^2 = x^4 + 12x^3 + 47x^2 + 72x + 36$$

$$(11) \quad (\text{与式}) = \{(x - 2)(x + 2)(x^2 + 4)\}^2 = \{(x^2 - 4)(x^2 + 4)\}^2 = (x^4 - 16)^2 = x^8 - 32x^4 + 256$$

$$(12) \quad (\text{与式}) = (x^2 - a^2)(x^2 + a^2)(x^4 + a^4) = (x^4 - a^4)(x^4 + a^4) = x^8 - a^8$$

9

解説

$$(1) \quad (2x - y)^2 + (x + 2y)(x - 2y) - (x - y)(5x + 3y) \\ = (4x^2 - 4xy + y^2) + (x^2 - 4y^2) - (5x^2 - 2xy - 3y^2) \\ = 4x^2 - 4xy + y^2 + x^2 - 4y^2 - 5x^2 + 2xy + 3y^2 \\ = -2xy$$

$$(2) \quad (a + 2)^2 - (a + 2)(b + 2) + a(b - 2) \\ = (a^2 + 4a + 4) - (ab + 2a + 2b + 4) + (ab - 2a) \\ = a^2 + 4a + 4 - ab - 2a - 2b - 4 + ab - 2a \\ = a^2 - 2b$$

$$(3) \quad (2a - b)(a + 2b) - (a + b)^2 + 3b^2 \\ = (2a^2 + 3ab - 2b^2) - (a^2 + 2ab + b^2) + 3b^2 \\ = 2a^2 + 3ab - 2b^2 - a^2 - 2ab - b^2 + 3b^2 \\ = a^2 + ab$$

$$(4) \quad (a + b + c)^2 - (a - b + c)^2 = \{(a + c) + b\}^2 - \{(a + c) - b\}^2$$

$$= \{(a + c)^2 + 2b(a + c) + b^2\} - \{(a + c)^2 - 2b(a + c) + b^2\} \\ = (a + c)^2 + 2b(a + c) + b^2 - (a + c)^2 + 2b(a + c) - b^2 \\ = 4b(a + c) \\ = 4ab + 4bc$$

$$(5) \quad (x + y + 1)(x + y - 1) - (x - y)^2 = \{(x + y) + 1\}[(x + y) - 1] - (x - y)^2 \\ = \{(x + y)^2 - 1^2\} - (x - y)^2 \\ = (x + y)^2 - 1 - (x - y)^2 \\ = (x^2 + 2xy + y^2) - 1 - (x^2 - 2xy + y^2) \\ = x^2 + 2xy + y^2 - 1 - x^2 + 2xy - y^2 \\ = 4xy - 1$$

$$(6) \quad (2x - 2y - 1)^2 - 4(x - y)^2 = [2(x - y) - 1]^2 - 4(x - y)^2 \\ = [4(x - y)^2 - 4(x - y) + 1^2] - 4(x - y)^2 \\ = 4(x - y)^2 - 4(x - y) + 1 - 4(x - y)^2 \\ = -4(x - y) + 1 \\ = -4x + 4y + 1$$

$$(7) \quad (a + b)^2 - (a - 1 + b)^2 = (a + b)^2 - \{(a + b) - 1\}^2 \\ = (a + b)^2 - \{(a + b)^2 - 2(a + b) + 1^2\} \\ = (a + b)^2 - (a + b)^2 + 2(a + b) - 1^2 \\ = 2(a + b) - 1 \\ = 2a + 2b - 1$$

10

解説

$$(1) \quad (2x - y)^2 + (x + y)(x + 3y) = (4x^2 - 4xy + y^2) + (x^2 + 4xy + 3y^2) \\ = 4x^2 - 4xy + y^2 + x^2 + 4xy + 3y^2 \\ = 5x^2 + 4y^2$$

$$= 5 \times (-4)^2 + 4 \times \left(\frac{3}{2}\right)^2 \\ = 80 + 9 \\ = 89$$

$$(2) \quad (x + y)(9x + y) - (3x - y)^2 = (9x^2 + 10xy + y^2) - (9x^2 - 6xy + y^2) \\ = 9x^2 + 10xy + y^2 - 9x^2 + 6xy - y^2 \\ = 16xy \\ = 16 \times (-4) \times \frac{3}{2} \\ = -96$$

11 [鎌倉学園]

解説

$$x^2 - xy + y^2 = (x^2 + 2xy + y^2) - 3xy \\ = (x + y)^2 - 3xy \\ \text{この式に } x + y = 3, \quad xy = 1 \text{ を代入すると} \\ (x + y)^2 - 3xy = 3^2 - 3 \times 1 \\ = 6$$

1

解説

$$(1) \quad \frac{(x - 2)(x - 3)}{4} - \frac{(x - 2)(x - 3) + 4}{5} = \frac{5(x - 2)(x - 3) - 4(x - 2)(x - 3) - 4 \times 4}{20} \\ = \frac{(x - 2)(x - 3) - 16}{20}$$

$$= \frac{(x^2 - 5x + 6) - 16}{20} \\ = \frac{x^2 - 5x - 10}{20}$$

$$(2) \quad \frac{(1 - x)(3x - 1)}{3} - \frac{(-3x + 2)(x + 1)}{4} = \frac{4(1 - x)(3x - 1) - 3(-3x + 2)(x + 1)}{12}$$

$$= \frac{4(-3x^2 + 4x - 1) - 3(-3x^2 - x + 2)}{12} \\ = \frac{-12x^2 + 16x - 4 + 9x^2 + 3x - 6}{12} \\ = \frac{-3x^2 + 19x - 10}{12}$$

$$(3) \quad \frac{(a - 2)^2}{2} - \frac{a(a - 1)}{4} + 2(a + 1)(a - 1) = \frac{2(a - 2)^2 - a(a - 1) + 8(a + 1)(a - 1)}{4} \\ = \frac{2(a^2 - 4a + 4) - (a^2 - a) + 8(a^2 - 1)}{4}$$

$$= \frac{2a^2 - 8a + 8 - a^2 + a + 8a^2 - 8}{4} \\ = \frac{9a^2 - 7a}{4}$$

$$(4) \quad \left(\frac{x - 1}{2}\right)^2 - \frac{(x - 2)(x + 3)}{4} + \frac{3}{2}x = \frac{(x - 1)^2}{4} - \frac{(x - 2)(x + 3)}{4} + \frac{3}{2}x \\ = \frac{(x - 1)^2 - (x - 2)(x + 3) + 3x \times 2}{4}$$

$$= \frac{x^2 - 2x + 1 - x^2 - x + 6 + 6x}{4} \\ = \frac{3x + 7}{4}$$

$$(5) \quad \frac{(x - 6y)(x + 2y)}{2} - \frac{(x - 3y)^2}{3} = \frac{3(x - 6y)(x + 2y) - 2(x - 3y)^2}{6} \\ = \frac{3(x^2 - 4xy - 12y^2) - 2(x^2 - 6xy + 9y^2)}{6}$$

$$= \frac{3x^2 - 12xy - 36y^2 - 2x^2 + 12xy - 18y^2}{6} \\ = \frac{x^2 - 54y^2}{6}$$

$$(6) \quad \frac{(x - 2y)^2}{3} - \frac{x(2x - 3y)}{4} + \frac{(7x - 4y)y}{12}$$

$$= \frac{4(x - 2y)^2 - 3x(2x - 3y) + (7x - 4y)y}{12} \\ = \frac{4(x^2 - 4xy + 4y^2) - 3x(2x - 3y) + (7x - 4y)y}{12} \\ = \frac{4x^2 - 16xy + 16y^2 - 6x^2 + 9xy + 7xy - 4y^2}{12} \\ = \frac{-2x^2 + 12y^2}{12} = \frac{-x^2 + 6y^2}{6}$$

2 [京都産業大]

解説

$$\begin{aligned}(x+y+z)(x-y-z) &= (x+(y+z))(x-(y+z)) \\ &= x^2 - (y+z)^2 \\ &= x^2 - (y^2 + 2yz + z^2) \\ &= x^2 - y^2 - z^2 - 2yz\end{aligned}$$

3

解説

$$\begin{aligned}(1) \quad & (x-y+1)(x-y-1) + (x+y-2)(x-y+2) \\ &= [(x-y)+1][(x-y)-1] + \{x+(y-2)\}(x-(y-2)) \\ &= \{(x-y)^2 - 1^2\} + \{x^2 - (y-2)^2\} \\ &= x^2 - 2xy + y^2 - 1 + x^2 - y^2 + 4y - 4 \\ &= 2x^2 - 2xy + 4y - 5 \\ (2) \quad & (x+y+z)^2 + (x+y-z)^2 + (y+z-x)^2 + (z+x-y)^2 \\ &= (x^2 + y^2 + z^2 + 2xy + 2yz + 2zx) + (x^2 + y^2 + z^2 + 2xy - 2yz - 2zx) \\ &\quad + (x^2 + y^2 + z^2 - 2xy + 2yz - 2zx) + (x^2 + y^2 + z^2 - 2xy - 2yz + 2zx) \\ &= 4x^2 + 4y^2 + 4z^2\end{aligned}$$

4 [函館大]

解説

$$\begin{aligned}(1) \quad & (x+y)(x-y)(x^2+y^2)(x^4+y^4) = (x^2-y^2)(x^2+y^2)(x^4+y^4) = (x^4-y^4)(x^4+y^4) \\ &= x^8 - y^8 \\ (2) \quad & (x-1)^2(x^2+x+1)^2 = \{(x-1)(x^2+x+1)\}^2 = (x^3-1)^2 = x^6 - 2x^3 + 1 \\ (3) \quad & (1+a)(1-a^3+a^6)(1-a+a^2) = \{(1+a)(1-a+a^2)\}(1-a^3+a^6) \\ &= (1+a^3)(1-a^3+a^6) \\ &= 1+a^9\end{aligned}$$

5

解説

$$\begin{aligned}(1) \quad & x(x+2)(x+3)(x+5) = \{x(x+5)\}\{(x+2)(x+3)\} \\ &= (x^2+5x)(x^2+5x+6) \\ &= (x^2+5x)\{(x^2+5x)+6\} \\ &= (x^2+5x)^2 + 6(x^2+5x) \\ &= x^4 + 10x^3 + 25x^2 + 6x^2 + 30x \\ &= x^4 + 10x^3 + 31x^2 + 30x \\ (2) \quad & (x-1)(x+1)(x+2)(x+4) = \{(x-1)(x+4)\}\{(x+1)(x+2)\} \\ &= (x^2+3x-4)(x^2+3x+2) \\ &= \{(x^2+3x)-4\}\{(x^2+3x)+2\} \\ &= (x^2+3x)^2 - 2(x^2+3x) - 8 \\ &= x^4 + 6x^3 + 9x^2 - 2x^2 - 6x - 8 \\ &= x^4 + 6x^3 + 7x^2 - 6x - 8 \\ (3) \quad & (x-1)(x-3)(x+1)(x+3) = \{(x-1)(x+1)\}\{(x-3)(x+3)\} \\ &= (x^2-1)(x^2-9) \\ &= x^4 - 10x^2 + 9 \\ (4) \quad & (x+2)(x+3)(x+4)(x+6) = \{(x+2)(x+6)\}\{(x+3)(x+4)\} \\ &= (x^2+8x+12)(x^2+7x+12)\end{aligned}$$

$$\begin{aligned}&= \{(x^2+12)+8x\}\{(x^2+12)+7x\} \\ &= (x^2+12)^2 + 15x(x^2+12) + 56x^2 \\ &= x^4 + 24x^2 + 144 + 15x^3 + 180x + 56x^2 \\ &= x^4 + 15x^3 + 80x^2 + 180x + 144\end{aligned}$$

6

解説

$$\begin{aligned}(1) \quad & x + \frac{1}{x} = 3 \text{ の両辺を } 2 \text{ 乗すると} \quad \left(x + \frac{1}{x}\right)^2 = 3^2 \\ & \text{よって} \quad x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 9 \\ \text{すなわち} \quad & x^2 + 2 + \frac{1}{x^2} = 9 \\ \text{したがって} \quad & x^2 + \frac{1}{x^2} = 7\end{aligned}$$

$$\begin{aligned}(2) \quad & x - \frac{1}{x} = 2 \text{ の両辺を } 2 \text{ 乗すると} \quad \left(x - \frac{1}{x}\right)^2 = 2^2 \\ & \text{よって} \quad x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 4 \\ \text{すなわち} \quad & x^2 - 2 + \frac{1}{x^2} = 4 \\ \text{したがって} \quad & x^2 + \frac{1}{x^2} = 6\end{aligned}$$

$$\begin{aligned}(3) \quad & x - \frac{1}{x} = 3 \text{ の両辺を } 2 \text{ 乗すると} \quad \left(x - \frac{1}{x}\right)^2 = 3^2 \\ & \text{よって} \quad x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 9 \\ \text{すなわち} \quad & x^2 + \frac{1}{x^2} = 11 \\ \text{したがって} \quad & x^2 - 3x + 2 + \frac{3}{x} + \frac{1}{x^2} = x^2 + \frac{1}{x^2} - 3\left(x - \frac{1}{x}\right) + 2 \\ &= 11 - 3 \times 3 + 2 = 4\end{aligned}$$

7

解説

$$\begin{aligned}(1) \quad & x^5 \text{ の項は} \quad x^3 \cdot 2x^2 + 3x^2 \cdot x^3 = 5x^5 \\ & x^3 \text{ の項は} \quad x^3 \cdot 1 + 3x^2 \cdot (-x) + 2x \cdot 2x^2 + 7 \cdot x^3 = 9x^3 \\ & \text{よって, } x^5 \text{ の係数は } 5 \quad x^3 \text{ の係数は } 9 \\ \text{【参考】} \quad & \text{すべて展開すると} \\ & \quad (\text{与式}) = x^6 + 5x^5 + 7x^4 + 9x^3 + 15x^2 - 5x + 7 \\ (2) \quad & \text{展開式において, } xyz \text{ の項が} 4 \text{ 出てくる計算だけを取り出すと} \\ & \quad 5x \cdot (-2yz) - y \cdot 10zx - 2z \cdot 5xy = -10xyz - 10xyz - 10xyz = -30xyz \\ & \text{よって, 求める係数は} \quad -30\end{aligned}$$

8

解説

$$\begin{aligned}(1) \quad & P = 2013^2 - 3 \times 2012^2 + 2 \times 2013 \times 2012 + 3 \times 2012 \times 2011 - 3 \times 2011 \times 2013 \\ &= (2012+1)^2 - 3 \times 2012^2 + 2 \times (2012+1) \times 2012 \\ &\quad + 3 \times 2012 \times (2012-1) - 3 \times (2012-1) \times (2012+1) \\ &= (a+1)^2 - 3a^2 + 2(a+1)a + 3a(a-1) - 3(a-1)(a+1) \\ &= a^2 + 2a + 1 - 3a^2 + 2a^2 + 2a + 3a^2 - 3a - 3a^2 + 3\end{aligned}$$

$$= a + 4$$

$$(2) \quad (1) \text{ から} \quad P = 2012 + 4 = 2016$$

9

解説

$$\begin{aligned}(1) \quad & 2083^2 - 2081 \times 2085 = 2083^2 - (2083-2) \times (2083+2) \\ &= 2083^2 - (2083^2 - 2^2) \\ &= 2^2 \\ &= 4 \\ (2) \quad & 5718 \times 5712 - 5715^2 = (5715+3) \times (5715-3) - 5715^2 \\ &= (5715^2 - 3^2) - 5715^2 \\ &= -3^2 \\ &= -9 \\ (3) \quad & 3916 \times 3912 - 3910 \times 3918 = (3914+2) \times (3914-2) - (3914-4) \times (3914+4) \\ &= (3914^2 - 2^2) - (3914^2 - 4^2) \\ &= -2^2 + 4^2 \\ &= 12 \\ (4) \quad & 4839 \times 4833 - 4840 \times 4832 = (4836+3) \times (4836-3) - (4836+4) \times (4836-4) \\ &= (4836^2 - 3^2) - (4836^2 - 4^2) \\ &= -3^2 + 4^2 \\ &= 7\end{aligned}$$

10 [星稜]

解説

大, 中, 小の3つの円において, 斜線部分は大円から小円と中円を除いた図形である。よって, 求める面積は

$$\begin{aligned}\pi \times \left(\frac{1}{2}(x+y)\right)^2 - \pi \times \left(\frac{1}{2}x\right)^2 - \pi \times \left(\frac{1}{2}y\right)^2 &= \frac{1}{4}\pi\{(x+y)^2 - x^2 - y^2\} \\ &= \frac{1}{4}\pi \times 2xy \\ &= \frac{1}{2}\pi xy \text{ (cm}^2\text{)}\end{aligned}$$

11

解説

$$\begin{aligned}T &= \frac{1}{2}\pi b^2 \\ S &= \frac{1}{2}\pi(a+b)^2 - \frac{1}{2}\pi a^2 - \frac{1}{2}\pi b^2 \\ &= \frac{1}{2}\pi(a^2 + 2ab + b^2 - a^2 - b^2) = \pi ab \\ S = 3T \text{ のとき} \quad & \pi ab = \frac{3}{2}\pi b^2 \\ \text{よって} \quad & a = \frac{3}{2}b \quad \text{したがって} \quad a : b = 3 : 2\end{aligned}$$

[12] [栃木県]

解説

中央の数を n とすると、連続する5つの整数は小さい方から $n-2$, $n-1$, n , $n+1$, $n+2$ と表すことができる。

よって

$$\begin{aligned}(n+2)(n+1)-(n-2)(n-1) &= n^2+3n+2-(n^2-3n+2) \\ &= n^2+3n+2-n^2+3n-2 \\ &= 6n\end{aligned}$$

よって、最も大きい数と2番目に大きい数の積から、最も小さい数と2番目に小さい数の積をひくと、中央の数の6倍になる。

[13] [香川県]

解説

整数 M の十の位の数を x 、一の位の数を y とすると

$$M=10x+y, N=x+y$$

と表すことができる。

$$\begin{aligned}\text{よって } M^2-N^2 &= (10x+y)^2-(x+y)^2 \\ &= 100x^2+20xy+y^2-(x^2+2xy+y^2) \\ &= 99x^2+18xy \\ &= 9(11x^2+2xy)\end{aligned}$$

$11x^2+2xy$ は整数であるから、 M^2-N^2 は9の倍数である。

[14] [福岡県]

解説

n を整数として、2番目に小さい数を $2n$ と表すと、最も小さい数は $2n-2$ 、2番目に大きい数は $2n+8$ 、最も大きい数は $2n+10$ と表される。

$$\begin{aligned}\text{よって } & \{(2n+10)+2n\}^2-\{(2n+8)+(2n-2)\}^2 \\ &= (4n+10)^2-(4n+6)^2 \\ &= 16n^2+80n+100-(16n^2+48n+36) \\ &= 32n+64 \\ &= 32(n+2)\end{aligned}$$

$n+2$ は整数であるから、 $32(n+2)$ は32でわりきれぬ。

したがって、4つの偶数において、最も大きい数と2番目に小さい数の和の2乗から、2番目に大きい数と最も小さい数の和の2乗をひいた差は、32でわりきれぬ。

[1]

解説

$$\begin{aligned}(1) \quad & (a+b-c)^2-(a-b+c)^2 \\ &= \{a+(b-c)\}^2-\{a-(b-c)\}^2 \\ &= a^2+2a(b-c)+(b-c)^2-\{a^2-2a(b-c)+(b-c)^2\} \\ &= a^2+2a(b-c)+(b-c)^2-a^2+2a(b-c)-(b-c)^2 \\ &= 4a(b-c)=4ab-4ac \\ (2) \quad & (x^2+y^2)^2+(x+y)^2(x-y)^2=(x^2+y^2)^2+\{(x+y)(x-y)\}^2 \\ &= (x^2+y^2)^2+(x^2-y^2)^2 \\ &= x^4+2x^2y^2+y^4+x^4-2x^2y^2+y^4 \\ &= 2x^4+2y^4 \\ (3) \quad & (a+b-c)(a-b+c)+(b-c)^2=\{a+(b-c)\}\{a-(b-c)\}+(b-c)^2 \\ &= a^2-(b-c)^2+(b-c)^2=a^2 \\ (4) \quad & (3x+y-z)^2-(3x+y)(3x+y-2z) \\ &= \{(3x+y)-z\}^2-(3x+y)\{(3x+y)-2z\} \\ &= (3x+y)^2-2(3x+y)z+z^2-(3x+y)^2+2(3x+y)z \\ &= z^2 \\ (5) \quad & (x+3)(x-5)(x+5)(x-3)=(x+3)(x-3)\times(x-5)(x+5) \\ &= (x^2-9)(x^2-25) \\ &= x^4-34x^2+225 \\ (6) \quad & (x-1)(x+1)(x^2+1)(x^4+1)=(x^2-1)(x^2+1)(x^4+1) \\ &= (x^4-1)(x^4+1)=x^8-1 \\ (7) \quad & (2x+1)(x+2)(2x-1)(x-2)=(2x+1)(2x-1)\times(x+2)(x-2) \\ &= (4x^2-1)(x^2-4)=4x^4-17x^2+4 \\ (8) \quad & (x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4) \\ &= \{(x^2+y^2)+xy\}\{(x^2+y^2)-xy\}(x^4-x^2y^2+y^4) \\ &= \{(x^2+y^2)^2-(xy)^2\}(x^4-x^2y^2+y^4) \\ &= (x^4+2x^2y^2+y^4-x^2y^2)(x^4-x^2y^2+y^4) \\ &= (x^4+x^2y^2+y^4)(x^4-x^2y^2+y^4) \\ &= \{(x^4+y^4)+x^2y^2\}\{(x^4+y^4)-x^2y^2\} \\ &= (x^4+y^4)^2-(x^2y^2)^2=x^8+2x^4y^4+y^8-x^4y^4 \\ &= x^8+x^4y^4+y^8 \\ (9) \quad & (x-3y)(x+3y)(x^2+3xy+y^2)(x^2-3xy+y^2) \\ &= (x^2-9y^2)\{(x^2+y^2)+3xy\}\{(x^2+y^2)-3xy\} \\ &= (x^2-9y^2)\{(x^2+y^2)^2-(3xy)^2\} \\ &= (x^2-9y^2)(x^4+2x^2y^2+y^4-9x^2y^2) \\ &= (x^2-9y^2)(x^4-7x^2y^2+y^4) \\ &= x^2(x^4-7x^2y^2+y^4)-9y^2(x^4-7x^2y^2+y^4) \\ &= x^6-7x^4y^2+x^2y^4-9x^4y^2+63x^2y^4-9y^6 \\ &= x^6-16x^4y^2+64x^2y^4-9y^6 \\ (10) \quad & (a+b+c)(-a+b+c)+(a-b+c)(a+b-c) \\ &= -\{a+(b+c)\}\{a-(b+c)\}+\{a-(b-c)\}\{a+(b-c)\} \\ &= -\{a^2-(b+c)^2\}+\{a^2-(b-c)^2\} \\ &= -a^2+(b+c)^2+a^2-(b-c)^2 \\ &= (b+c)^2-(b-c)^2\end{aligned}$$

$$\begin{aligned}&= b^2+2bc+c^2-(b^2-2bc+c^2) \\ &= 4bc\end{aligned}$$

$$\begin{aligned}(11) \quad & (a+b+c)^2-(a-b-c)(a+b-c)-2(b+c)(a+b) \\ &= \{(a+b+c)^2-\{(a-c)-b\}\{(a-c)+b\}-2(b+c)(b+a) \\ &= (a+b)^2+2(a+b)c+c^2-\{(a-c)^2-b^2\}-2\{b^2+(c+a)b+ca\} \\ &= a^2+2ab+b^2+2ac+2bc+c^2-(a^2-2ac+c^2-b^2)-2\{b^2+bc+ab+ca\} \\ &= 2ca \\ (12) \quad & \text{与式}=\{(a+b)^2-1\}\{(a-b)^2-1\} \\ &= (a^2+2ab+b^2-1)(a^2-2ab+b^2-1) \\ &= \{(a^2+b^2-1)+2ab\}\{(a^2+b^2-1)-2ab\} \\ &= (a^2+b^2-1)^2-(2ab)^2 \\ &= a^4+b^4+1+2a^2b^2-2b^2-2a^2-4a^2b^2 \\ &= a^4+b^4-2a^2b^2-2a^2-2b^2+1\end{aligned}$$

[2] [桐光学園]

解説

$x+y=1$ の両辺を2乗すると

$$\begin{aligned}(x+y)^2 &= 1^2 \\ x^2+2xy+y^2 &= 1 \\ x^2+y^2+2xy &= 1\end{aligned}$$

これに $x^2+y^2=5$ を代入して

$$\begin{aligned}5+2xy &= 1 \\ 2xy &= -4 \\ xy &= -2\end{aligned}$$

$$\begin{aligned}\text{よって } \frac{1}{x}+\frac{1}{y} &= \frac{y+x}{xy}=\frac{x+y}{xy} \\ &= -\frac{1}{2}\end{aligned}$$

[3]

解説

$$\begin{aligned}(1) \quad & (a+b)(c+d)=ac+ad+bc+bd \\ &= ac+1+1+bd \\ &= ac+bd+2\end{aligned}$$

ここで、 $(a+b)(c+d)=5\times 2$ であるから

$$\begin{aligned}ac+bd+2 &= 10 \\ ac+bd &= 8\end{aligned}$$

$$(2) \quad b=5-a, d=2-c \text{ を } ad=bc \text{ に代入して}$$

$$\begin{aligned}a(2-c) &= (5-a)c \\ 2a-ac &= 5c-ac \\ 2a &= 5c \\ \frac{a}{c} &= \frac{5}{2}\end{aligned}$$

[4] [金沢工業大]

解説

$$x^2+y^2+z^2=(x+y+z)^2-2(xy+yz+zx)=7^2-2\cdot 14=21$$

5 [東北学院大]

解説

$$\frac{1}{1-a} + \frac{1}{1+a} = \frac{1+a+1-a}{(1-a)(1+a)} = \frac{2}{1-a^2},$$

$$\frac{2}{1-a^2} + \frac{2}{1+a^2} = 2 \cdot \frac{1+a^2+1-a^2}{(1-a^2)(1+a^2)} = \frac{4}{1-a^4},$$

$$\frac{4}{1-a^4} + \frac{4}{1+a^4} = 4 \cdot \frac{1+a^4+1-a^4}{(1-a^4)(1+a^4)} = \frac{8}{1-a^8}$$

$$\begin{aligned} \text{よって} \quad (\text{与式}) &= \frac{8}{1-a^8} + \frac{8}{1+a^8} \\ &= 8 \cdot \frac{1+a^8+1-a^8}{(1-a^8)(1+a^8)} = \frac{16}{1-a^{16}} \end{aligned}$$

6

解説

小さい方の整数を 5 でわったときの商を n (n は 0 以上の整数) とすると、

$$\text{小さい方の整数は} \quad 5 \times n + 2 = 5n + 2$$

$$\text{大きい方の整数は} \quad 5n + 3$$

と表される。

この 2 つの整数の積は

$$\begin{aligned} (5n+2)(5n+3) &= 25n^2 + 25n + 6 \\ &= 25n^2 + 25n + 5 + 1 \\ &= 5(5n^2 + 5n + 1) + 1 \end{aligned}$$

$5n^2 + 5n + 1$ は整数であるから、5 でわったときの余りは 1 である。

7 [大分県]

解説

(1) 例えば、2 つの自然数を 1 と 3 とすると

$$A = 1 + 3 = 4$$

$$B = 3 - 1 = 2$$

$$\text{このとき} \quad X = 4^2 - 2^2 = 12$$

X は 8 の倍数ではない。

$$\text{よって} \quad (\text{ア}) \quad 1 \quad (\text{イ}) \quad 3 \quad (\text{ウ}) \quad 12$$

(2) $A = a + b$, $B = a - b$ である。

$$\begin{aligned} \text{このとき} \quad X &= (a+b)^2 - (a-b)^2 \\ &= a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) \\ &= 4ab \end{aligned}$$

a , b は自然数であるから、 $4ab$ は 4 の倍数である。

したがって、 X はいつでも 4 の倍数である。

(3) (2) より $4ab = 2016$

$$ab = 504$$

この式を満たす a , b のうち、 $a - b$ の値がもっとも小さくなるものをみつける。

$504 = 2^3 \times 3^2 \times 7$ であるから、求める 2 つの自然数は

$$21, 24$$