数Ⅲ積分　　チェックテスト　【解答】

各10点

解答　(1) $\dfrac{\pi}{16}-\dfrac{1}{8}$　(2) $\dfrac{\sqrt{2}}{5}$　(3) 2　(4) $\dfrac{1}{2}+\log 2$　(5) $\log\dfrac{3}{2}$

(6) $16\log 2-9\log 3$　(7) $\log(e+1)$　(8) $\dfrac{14}{15}$　(9) $\dfrac{\pi}{2}-1$　(10) $\dfrac{\pi}{4}$

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(解説)

(1) $\displaystyle\int_0^{\frac{\pi}{8}}\sin^2 2x\,dx=\int_0^{\frac{\pi}{8}}\frac{1-\cos 4x}{2}dx=\frac{1}{2}\left[x-\frac{\sin 4x}{4}\right]_0^{\frac{\pi}{8}}$

$\displaystyle=\frac{1}{2}\left(\frac{\pi}{8}-\frac{1}{4}\right)=\frac{\pi}{16}-\frac{1}{8}$

(2) $\displaystyle\int_0^{\frac{\pi}{4}}\cos 2x\cos 3x\,dx=\int_0^{\frac{\pi}{4}}\frac{1}{2}(\cos 5x+\cos x)dx=\frac{1}{2}\left[\frac{1}{5}\sin 5x+\sin x\right]_0^{\frac{\pi}{4}}$

$\displaystyle=\frac{1}{2}\left(-\frac{\sqrt{2}}{10}+\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{5}$

(3) $0\leqq x\leqq\frac{\pi}{3}$ のとき　$\displaystyle\left|\sin\left(x-\frac{\pi}{3}\right)\right|=-\sin\left(x-\frac{\pi}{3}\right)$

$\frac{\pi}{3}\leqq x\leqq\pi$ のとき　$\displaystyle\left|\sin\left(x-\frac{\pi}{3}\right)\right|=\sin\left(x-\frac{\pi}{3}\right)$

よって　$\displaystyle I=-\int_0^{\frac{\pi}{3}}\sin\left(x-\frac{\pi}{3}\right)dx+\int_{\frac{\pi}{3}}^{\pi}\sin\left(x-\frac{\pi}{3}\right)dx$

$\displaystyle=\left[\cos\left(x-\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{3}}-\left[\cos\left(x-\frac{\pi}{3}\right)\right]_{\frac{\pi}{3}}^{\pi}$

$\displaystyle=\left\{\cos 0-\cos\left(-\frac{\pi}{3}\right)\right\}-\left(\cos\frac{2}{3}\pi-\cos 0\right)$

$\displaystyle=2\cdot 1-\frac{1}{2}-\left(-\frac{1}{2}\right)=2$

(4) $\displaystyle\int_0^1\frac{x^2+x+1}{x+1}dx=\int_0^1\left(x+\frac{1}{x+1}\right)dx=\left[\frac{x^2}{2}+\log(x+1)\right]_0^1$

$\displaystyle=\frac{1}{2}+\log 2$

(5) $\displaystyle\int_1^3\frac{dx}{x(x+1)}=\int_1^3\left(\frac{1}{x}-\frac{1}{x+1}\right)dx=\left[\log\frac{x}{x+1}\right]_1^3$

$\displaystyle=\log\frac{3}{4}-\log\frac{1}{2}=\log\frac{3}{2}$

(6) $\displaystyle\frac{2x+1}{x^2-7x+12}=\frac{2x+1}{(x-3)(x-4)}=-\frac{7}{x-3}+\frac{9}{x-4}$

よって

$\displaystyle\int_1^2\frac{2x+1}{x^2-7x+12}dx=\int_1^2\left(-\frac{7}{x-3}+\frac{9}{x-4}\right)dx=\Big[-7\log|x-3|+9\log|x-4|\Big]_1^2$

$=16\log 2-9\log 3$

(7) $\displaystyle\int_1^2\frac{e^x}{e^x-1}dx=\int_1^2\frac{(e^x-1)'}{e^x-1}dx=\Big[\log|e^x-1|\Big]_1^2=\log(e^2-1)-\log(e-1)$

$=\log\dfrac{e^2-1}{e-1}=\log(e+1)$

(8)　$\sqrt{2-x}=t$ とおくと　　$x=2-t^2$，$dx=-2tdt$

x	$1 \longrightarrow 2$
t	$1 \longrightarrow 0$

よって　　$\displaystyle\int_1^2 x\sqrt{2-x}\,dx=\int_1^0(2-t^2)t\cdot(-2t)dt=2\int_0^1(2t^2-t^4)dt$

$\displaystyle=2\left[\frac{2}{3}t^3-\frac{t^5}{5}\right]_0^1=2\left(\frac{2}{3}-\frac{1}{5}\right)=\frac{14}{15}$

(9)　$\displaystyle\int_0^{\frac{\pi}{2}}x\cos x\,dx=\Big[x\sin x\Big]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\sin x\,dx=\frac{\pi}{2}+\Big[\cos x\Big]_0^{\frac{\pi}{2}}=\frac{\pi}{2}-1$

(10)　$x=\sin\theta$ とおくと　$dx=\cos\theta\,d\theta$

ゆえに　(与式)$\displaystyle=\int_0^{\frac{\pi}{2}}\sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta=\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta=\int_0^{\frac{\pi}{2}}\frac{1+\cos2\theta}{2}d\theta$

$\displaystyle=\left[\frac{1}{2}\theta+\frac{1}{4}\sin2\theta\right]_0^{\frac{\pi}{2}}=\frac{\pi}{4}$