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- 【解答】 (1) $xy+3x+2y+6$ (2) $xy-2x+7y-14$ (3) $ab+8a-4b-32$
 (4) $kl-7k-11l+77$ (5) $ac+3ad-2bc-6bd$
 (6) $px-13py+5qx-65qy$ (7) $2x^2+7x+3$
 (8) $2a^2+7a-15$ (9) $56n^2-19n-15$
 (10) $20x^2+3xy-2y^2$ (11) $8m^2+30mn-27n^2$

【解説】

- (1) $(x+2)(y+3)=xy+3x+2y+6$
 (2) $(x+7)(y-2)=xy-2x+7y-14$
 (3) $(a-4)(b+8)=ab+8a-4b-32$
 (4) $(k-11)(l-7)=kl-7k-11l+77$
 (5) $(a-2b)(c+3d)=ac+3ad-2bc-6bd$
 (6) $(p+5q)(x-13y)=px-13py+5qx-65qy$
 (7) $(2x+1)(x+3)=2x^2+6x+x+3=2x^2+7x+3$
 (8) $(2a-3)(a+5)=2a^2+10a-3a-15=2a^2+7a-15$
 (9) $(8n+3)(7n-5)=56n^2-40n+21n-15=56n^2-19n-15$
 (10) $(4x-y)(5x+2y)=20x^2+8xy-5xy-2y^2=20x^2+3xy-2y^2$
 (11) $(9m+2m)(4m-3n)=36mn-27n^2+8m^2-6mn=8m^2+30mn-27n^2$

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- 【解答】 (1) x^3-2x^2-x+2 (2) $-2x^4+3x^3+3x^2-x+6$
 (3) $3a^2-ab-4b^2+2a+2b$ (4) $8x^2-6xy+y^2+4x-2y$
 (5) $15a^2-2ab-8b^2-9a-6b$ (6) $28p^2+51pq-27q^2-22p+63q-30$
 (7) $3m^2-4mn-4n^2+5m+14n-12$
 (8) $3x^3-10x^2y+6xy^2-y^3$ (9) $2a^3-9a^2b+17ab^2-12b^3$
 (10) $2x^4-3x^3y-23x^2y^2-3xy^3+20y^4$

【解説】

- (1) $(x+1)(x^2-3x+2)=x(x^2-3x+2)+(x^2-3x+2)$
 $=x^3-3x^2+2x+x^2-3x+2$
 $=x^3-2x^2-x+2$
 (2) $(2-x+2x^2)(3+x-x^2)=(2x^2-x+2)(-x^2+x+3)$
 $=2x^2(-x^2+x+3)-x(-x^2+x+3)+2(-x^2+x+3)$
 $=-2x^4+2x^3+6x^2+x^3-x^2-3x-2x^2+2x+6$
 $=-2x^4+3x^3+3x^2-x+6$
 (3) $(a+b)(3a-4b+2)=a(3a-4b+2)+b(3a-4b+2)$
 $=3a^2-4ab+2a+3ab-4b^2+2b$
 $=3a^2-ab-4b^2+2a+2b$
 (4) $(4x-y+2)(2x-y)=4x(2x-y)-y(2x-y)+2(2x-y)$
 $=8x^2-4xy-2xy+y^2+4x-2y$
 $=8x^2-6xy+y^2+4x-2y$
 (5) $(3a+2b)(5a-4b-3)=3a(5a-4b-3)+2b(5a-4b-3)$
 $=15a^2-12ab-9a+10ab-8b^2-6b$
 $=15a^2-2ab-8b^2-9a-6b$
 (6) $(7p-3q+5)(4p+9q-6)=7p(4p+9q-6)-3q(4p+9q-6)+5(4p+9q-6)$

- $=28p^2+63pq-42p-12pq-27q^2+18q+20p+45q-30$
 $=28p^2+51pq-27q^2-22p+63q-30$
 (7) $(3m+2n-4)(m-2n+3)=3m(m-2n+3)+2n(m-2n+3)-4(m-2n+3)$
 $=3m^2-6mn+9m+2mn-4n^2+6n-4m+8n-12$
 $=3m^2-4mn-4n^2+5m+14n-12$
 (8) $(x^2-3xy+y^2)(3x-y)=x^2(3x-y)-3xy(3x-y)+y^2(3x-y)$
 $=3x^3-x^2y-9x^2y+3xy^2+3xy^2-y^3$
 $=3x^3-10x^2y+6xy^2-y^3$
 (9) $(2a-3b)(a^2-3ab+4b^2)=2a(a^2-3ab+4b^2)-3b(a^2-3ab+4b^2)$
 $=2a^3-6a^2b+8ab^2-3a^2b+9ab^2-12b^3$
 $=2a^3-9a^2b+17ab^2-12b^3$
 (10) $(3xy+2x^2-4y^2)(x^2-5y^2-3xy)$
 $= (2x^2+3xy-4y^2)(x^2-3xy-5y^2)$
 $=2x^2(x^2-3xy-5y^2)+3xy(x^2-3xy-5y^2)-4y^2(x^2-3xy-5y^2)$
 $=2x^4-6x^3y-10x^2y^2+3x^3y-9x^2y^2-15xy^3-4x^2y^2+12xy^3+20y^4$
 $=2x^4-3x^3y-23x^2y^2-3xy^3+20y^4$

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- 【解答】 (1) $x^2+7x+12$ (2) $x^2+14x+45$ (3) $a^2+11a+24$ (4) $x^2-4x-12$
 (5) $x^2-5x-24$ (6) $p^2+3p-70$ (7) x^2+4x-5 (8) $x^2-6x-72$
 (9) $y^2+7y-120$ (10) $x^2-12x+27$ (11) $x^2-17x+52$
 (12) $a^2-18a+77$

【解説】

- (1) $(x+3)(x+4)=x^2+(3+4)x+3\times4=x^2+7x+12$
 (2) $(x+9)(x+5)=x^2+(9+5)x+9\times5=x^2+14x+45$
 (3) $(a+3)(a+8)=a^2+(3+8)a+3\times8=a^2+11a+24$
 (4) $(x-6)(x+2)=x^2+(-6+2)x+(-6)\times2=x^2-4x-12$
 (5) $(x-8)(x+3)=x^2+(-8+3)x+(-8)\times3=x^2-5x-24$
 (6) $(p-7)(p+10)=p^2+(-7+10)p+(-7)\times10=p^2+3p-70$
 (7) $(x+5)(x-1)=x^2+(5-1)x+5\times(-1)=x^2+4x-5$
 (8) $(x+6)(x-12)=x^2+(6-12)x+6\times(-12)=x^2-6x-72$
 (9) $(y+15)(y-8)=y^2+(15-8)y+15\times(-8)=y^2+7y-120$
 (10) $(x-9)(x-3)=x^2+(-9-3)x+(-9)\times(-3)=x^2-12x+27$
 (11) $(x-4)(x-13)=x^2+(-4-13)x+(-4)\times(-13)=x^2-17x+52$
 (12) $(a-11)(a-7)=a^2+(-11-7)a+(-11)\times(-7)=a^2-18a+77$

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- 【解答】 (1) $4x^2+8x+3$ (2) $16a^2-16a-21$ (3) $9m^2+9m-4$
 (4) $64y^2-48y+5$ (5) $25x^2-20x-12$ (6) $49x^2+42x-27$

【解説】

- (1) $(2x+3)(2x+1)=(2x)^2+(3+1)\times2x+3\times1=4x^2+8x+3$
 (2) $(4a+3)(4a-7)=(4a)^2+(3-7)\times4a+3\times(-7)=16a^2-16a-21$
 (3) $(3m-1)(3m+4)=(3m)^2+(-1+4)\times3m+(-1)\times4=9m^2+9m-4$
 (4) $(8y-1)(8y-5)=(8y)^2+(-1-5)\times8y+(-1)\times(-5)=64y^2-48y+5$
 (5) $(5x+2)(5x-6)=(5x)^2+(2-6)\times5x+2\times(-6)=25x^2-20x-12$

- (6) $(7x-3)(7x+9)=(7x)^2+(-3+9)\times7x+(-3)\times9=49x^2+42x-27$

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- 【解答】 (1) $x^2+16x+64$ (2) $x^2-18x+81$ (3) $25y^2+20y+4$
 (4) $4a^2-28a+49$ (5) $x^2-6xy+9y^2$ (6) $\frac{1}{36}a^2+3ab+81b^2$
 (7) a^2-4a+4 (8) $9m^2-12mn+4n^2$ (9) $4p^2+4pq+q^2$

【解説】

- (1) $(x+8)^2=x^2+2\times8\times x+8^2=x^2+16x+64$
 (2) $(x-9)^2=x^2-2\times9\times x+9^2=x^2-18x+81$
 (3) $(5y+2)^2=(5y)^2+2\times5y\times2+2^2=25y^2+20y+4$
 (4) $(2a-7)^2=(2a)^2-2\times2a\times7+7^2=4a^2-28a+49$
 (5) $(x-3y)^2=x^2-2\times x\times3y+(3y)^2=x^2-6xy+9y^2$

$$(6) \left(\frac{1}{6}a+9b\right)^2=\left(\frac{1}{6}a\right)^2+2\times\frac{1}{6}a\times9b+(9b)^2=\frac{1}{36}a^2+3ab+81b^2$$

$$(7) (-a+2)^2=(-a)^2+2\times(-a)\times2+2^2=a^2-4a+4$$

$$\text{【別解】 } (-a+2)^2=-(-a-2)^2=(a-2)^2$$

$$=a^2-2\times2\times a+2^2=a^2-4a+4$$

$$(8) (-3m+2n)^2=(-3m)^2+2\times(-3m)\times2n+(2n)^2=9m^2-12mn+4n^2$$

$$\text{【別解】 } (-3m+2n)^2=[-(3m-2n)]^2=(3m-2n)^2$$

$$=(3m)^2-2\times3m\times2n+(2n)^2=9m^2-12mn+4n^2$$

$$(9) (-2p-q)^2=(-2p)^2-2\times(-2p)\times q+q^2=4p^2+4pq+q^2$$

$$\text{【別解】 } (-2p-q)^2=-(-2p+q)^2=(2p+q)^2$$

$$=(2p)^2+2\times2p\times q+q^2=4p^2+4pq+q^2$$

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- 【解答】 (1) x^2-4 (2) $49x^2-4y^2$ (3) $9-25x^2$ (4) $9y^2-4x^2$
 (5) a^2-25 (6) $\frac{1}{4}-\frac{4}{25}a^2$

【解説】

- (1) $(x+2)(x-2)=x^2-2^2=x^2-4$
 (2) $(7x-2y)(7x+2y)=(7x)^2-(2y)^2=49x^2-4y^2$
 (3) $(3-5x)(5x+3)=(3-5x)(3+5x)=3^2-(5x)^2=9-25x^2$
 (4) $(-2x+3y)(3y+2x)=(3y-2x)(3y+2x)=(3y)^2-(2x)^2=9y^2-4x^2$

$$(5) (-5-a)(5-a)=(-a-5)(-a+5)=(-a)^2-5^2=a^2-25$$

$$(6) \left(-\frac{2}{5}a+\frac{1}{2}\right)\left(\frac{1}{2}+\frac{2}{5}a\right)=\left(\frac{1}{2}-\frac{2}{5}a\right)\left(\frac{1}{2}+\frac{2}{5}a\right)$$

$$=\left(\frac{1}{2}\right)^2-\left(\frac{2}{5}a\right)^2=\frac{1}{4}-\frac{4}{25}a^2$$

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- 【解答】 (1) $2x^2-7$ (2) $3x+27$ (3) $8x^2-9x+8$ (4) $8m^2+18n^2$

中1 甲陽エンリート数学 授業問題No6 ～展開・因数分解・平方根～【解答&解説】

$$(5) \quad -8x^2 + 7xy - 3y^2 \quad (6) \quad 3a^2 + 4a - 7 \quad (7) \quad 3x - 27$$

$$(8) \quad 20a^2 + 30ab - 50b^2$$

【解説】

$$(1) \quad (x+2)(x-8) + (x+3)^2 = (x^2 - 6x - 16) + (x^2 + 6x + 9)$$

$$= x^2 - 6x - 16 + x^2 + 6x + 9$$

$$= 2x^2 - 7$$

$$(2) \quad (x-3)(x+5) - (x+6)(x-7) = (x^2 + 2x - 15) - (x^2 - x - 42)$$

$$= x^2 + 2x - 15 - x^2 + x + 42$$

$$= 3x + 27$$

$$(3) \quad (3x-2)^2 - (x+1)(x-4) = (9x^2 - 12x + 4) - (x^2 - 3x - 4)$$

$$= 9x^2 - 12x + 4 - x^2 + 3x + 4$$

$$= 8x^2 - 9x + 8$$

$$(4) \quad (2m+3n)^2 + (2m-3n)^2 = (4m^2 + 12mn + 9n^2) + (4m^2 - 12mn + 9n^2)$$

$$= 4m^2 + 12mn + 9n^2 + 4m^2 - 12mn + 9n^2$$

$$= 8m^2 + 18n^2$$

$$(5) \quad (x+2y)(x-y) - (3x-y)^2 = (x^2 + xy - 2y^2) - (9x^2 - 6xy + y^2)$$

$$= x^2 + xy - 2y^2 - 9x^2 + 6xy - y^2$$

$$= -8x^2 + 7xy - 3y^2$$

$$(6) \quad (-a+3)(-a-3) + 2(a+1)^2 = (a^2 - 9) + 2(a^2 + 2a + 1)$$

$$= a^2 - 9 + 2a^2 + 4a + 2$$

$$= 3a^2 + 4a - 7$$

$$(7) \quad (x+3)(x-7) + (x-3)^2 - (2x-3)(x-5)$$

$$= (x^2 - 4x - 21) + (x^2 - 6x + 9) - (2x^2 - 13x + 15)$$

$$= x^2 - 4x - 21 + x^2 - 6x + 9 - 2x^2 + 13x - 15$$

$$= 3x - 27$$

$$(8) \quad (5a+3b)(5a-3b) - (2a-7b)^2 - (a-4b)(a+2b)$$

$$= (25a^2 - 9b^2) - (4a^2 - 28ab + 49b^2) - (a^2 - 2ab - 8b^2)$$

$$= 25a^2 - 9b^2 - 4a^2 + 28ab - 49b^2 - a^2 + 2ab + 8b^2$$

$$= 20a^2 + 30ab - 50b^2$$

【8】

【解答】 (1) $\frac{2x^2 + 3x + 15}{6}$ (2) $\frac{10a^2 - 15b^2}{12}$ (3) $\frac{2a^2 - ab + 10b^2}{6}$

(4) $\frac{x^2 - y^2}{6}$

【解説】

$$(1) \quad \frac{(x+3)^2}{3} - \frac{3x+1}{2} = \frac{2(x+3)^2 - 3(3x+1)}{6}$$

$$= \frac{2(x^2 + 6x + 9) - 3(3x+1)}{6}$$

$$= \frac{2x^2 + 12x + 18 - 9x - 3}{6}$$

$$= \frac{2x^2 + 3x + 15}{6}$$

$$(2) \quad \frac{(2a-b)^2}{4} - \frac{(a-3b)^2}{6} = \frac{3(2a-b)^2 - 2(a-3b)^2}{12}$$

$$= \frac{3(4a^2 - 4ab + b^2) - 2(a^2 - 6ab + 9b^2)}{12}$$

$$= \frac{12a^2 - 12ab + 3b^2 - 2a^2 + 12ab - 18b^2}{12}$$

$$= \frac{10a^2 - 15b^2}{12}$$

$$(3) \quad \frac{2a^2 + 3b^2}{4} - \frac{(a-2b)(a+3b)}{6} - \frac{b^2}{12}$$

$$= \frac{3(2a^2 + 3b^2) - 2(a-2b)(a+3b) - b^2}{12}$$

$$= \frac{3(2a^2 + 3b^2) - 2(a^2 + ab - 6b^2) - b^2}{12}$$

$$= \frac{6a^2 + 9b^2 - 2a^2 - 2ab + 12b^2 - b^2}{12}$$

$$= \frac{4a^2 - 2ab + 20b^2}{12}$$

$$= \frac{2(2a^2 - ab + 10b^2)}{12} = \frac{2a^2 - ab + 10b^2}{6}$$

$$(4) \quad \frac{(x-y)^2}{2} - \frac{(x+y)(x-y)}{3} + (x-y)y$$

$$= \frac{3(x-y)^2 - 2(x+y)(x-y) + 6(x-y)y}{6}$$

$$= \frac{3(x^2 - 2xy + y^2) - 2(x^2 - y^2) + 6(xy - y^2)}{6}$$

$$= \frac{3x^2 - 6xy + 3y^2 - 2x^2 + 2y^2 + 6xy - 6y^2}{6}$$

$$= \frac{x^2 - y^2}{6}$$

【9】

【解答】 (1) $-12a^3 + 8a^2b^2$ (2) $x^2 - 8xy$ (3) $x^3 + 2x^2$ (4) $6xy$

(5) $a^2 - 12a$ (6) $7x^2 - 5xy$ (7) $-36p^2 + 6pq - 4q^2$ (8) $137x^2 - 90x$

【解説】

$$(1) \quad (9a^2b - 6ab^3) \div \left(-\frac{3}{2}ab\right) \times 2a^2 = (9a^2b - 6ab^3) \times \left(-\frac{2}{3ab}\right) \times 2a^2$$

$$= -\frac{36a^4b}{3ab} + \frac{24a^3b^3}{3ab}$$

$$= -12a^3 + 8a^2b^2$$

$$(2) \quad 5x(x-y) - (4x^3y + 3x^2y^2) \div xy = 5x(x-y) - (4x^3y + 3x^2y^2) \times \frac{1}{xy}$$

$$= 5x^2 - 5xy - \frac{4x^3y}{xy} - \frac{3x^2y^2}{xy}$$

$$= 5x^2 - 5xy - 4x^2 - 3xy$$

$$= x^2 - 8xy$$

$$(3) \quad \left(\frac{x^4}{2} - \frac{x^3}{3}\right) \div \frac{x}{6} + (4x^2 - 8x) \times \left(-\frac{x}{2}\right) = \left(\frac{x^4}{2} - \frac{x^3}{3}\right) \times \frac{6}{x} + (4x^2 - 8x) \times \left(-\frac{x}{2}\right)$$

$$= \frac{6x^4}{2x} - \frac{6x^3}{3x} - \frac{4x^3}{2} + \frac{8x^2}{2}$$

$$= 3x^3 - 2x^2 - 2x^2 + 4x^2$$

$$= x^3 + 2x^2$$

$$(4) \quad (6x^2y + 2x^2y^3) \div 2xy - x(3-y)^2 + 6x = (6x^2y + 2x^2y^3) \times \frac{1}{2xy} - x(3-y)^2 + 6x$$

$$= \frac{6x^2y}{2xy} + \frac{2x^2y^3}{2xy} - x(y^2 - 6y + 9) + 6x$$

$$= 3x + xy^2 - xy^2 + 6xy - 9x + 6x$$

$$= 6xy$$

$$(5) \quad 2a^2 + 3a - a[4a - 3(a-5)] = 2a^2 + 3a - a(4a - 3a + 15)$$

$$= 2a^2 + 3a - a(a + 15)$$

$$= 2a^2 + 3a - a^2 - 15a$$

$$= a^2 - 12a$$

$$(6) \quad x^2 - 3xy - 2x(x+4y - (4x+3y)) = x^2 - 3xy - 2x(x+4y-4x-3y)$$

$$= x^2 - 3xy - 2x(-3x+y)$$

$$= x^2 - 3xy + 6x^2 - 2xy$$

$$= 7x^2 - 5xy$$

$$(7) \quad -2\{3p^2 - 5p(2q-3p)\} - 2q(2q+7p) = -2\{3p^2 - 10pq + 15p^2\} - 4q^2 - 14pq$$

$$= -2(18p^2 - 10pq) - 4q^2 - 14pq$$

$$= -36p^2 + 20pq - 4q^2 - 14pq$$

$$= -36p^2 + 6pq - 4q^2$$

$$(8) \quad 5x^2 - 3\{6x - 4[2x^2 - 3x(2-3x)]\} = 5x^2 - 3\{6x - 4(2x^2 - 6x + 9x^2)\}$$

$$= 5x^2 - 3\{6x - 4(11x^2 - 6x)\}$$

$$= 5x^2 - 3\{6x - 44x^2 + 24x\}$$

$$= 5x^2 - 3(-44x^2 + 30x)$$

$$= 5x^2 + 132x^2 - 90x$$

$$= 137x^2 - 90x$$

【10】

【解答】 (1) $a^2 - 2ab + b^2 + 2a - 2b + 1$ (2) $4a^2 + b^2 + c^2 - 4ab + 2bc - 4ca$

(3) $a^2 + 4b^2 + 9c^2 + 4ab + 12bc + 6ca$

(4) $9x^2 + 25y^2 + 16z^2 + 30xy + 40yz + 24zx$

(5) $81x^2 + 64y^2 + 36z^2 + 144xy - 96yz - 108zx$

(6) $49x^2 + 25y^2 + 9z^2 - 70xy - 30yz + 42zx$

【解説】

$$(1) \quad (a-b+1)^2 = [(a-b)+1]^2$$

$$= (a-b)^2 + 2(a-b) \times 1 + 1^2$$

$$= a^2 - 2ab + b^2 + 2a - 2b + 1$$

$$(2) \quad (2a-b-c)^2 = [(2a-b)-c]^2$$

$$= (2a-b)^2 - 2(2a-b)c + c^2$$

$$= 4a^2 - 4ab + b^2 - 4ac + 2bc + c^2$$

$$= 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ca$$

$$(3) \quad (a+2b+3c)^2 = [(a+2b)+3c]^2$$

$$= (a+2b)^2 + 2(a+2b) \times 3c + (3c)^2$$

$$= a^2 + 4ab + 4b^2 + 6ac + 12bc + 9c^2$$

$$= a^2 + 4b^2 + 9c^2 + 4ab + 12bc + 6ca$$

$$\begin{aligned}(4) \quad (3x+5y+4z)^2 &= ((3x+5y)+4z)^2 \\ &= (3x+5y)^2 + 2(3x+5y) \times 4z + (4z)^2 \\ &= 9x^2 + 30xy + 25y^2 + 24xz + 40yz + 16z^2 \\ &= 9x^2 + 25y^2 + 16z^2 + 30xy + 40yz + 24zx\end{aligned}$$

$$\begin{aligned}(5) \quad (9x+8y-6z)^2 &= ((9x+8y)-6z)^2 \\ &= (9x+8y)^2 - 2(9x+8y) \times 6z + (6z)^2 \\ &= 81x^2 + 144xy + 64y^2 - 108xz - 96yz + 36z^2 \\ &= 81x^2 + 64y^2 + 36z^2 + 144xy - 96yz - 108zx\end{aligned}$$

$$\begin{aligned}(6) \quad (-7x+5y-3z)^2 &= ((-7x+5y)-3z)^2 \\ &= (-7x+5y)^2 - 2(-7x+5y) \times 3z + (3z)^2 \\ &= 49x^2 - 70xy + 25y^2 + 42xz - 30yz + 9z^2 \\ &= 49x^2 + 25y^2 + 9z^2 - 70xy - 30yz + 42zx\end{aligned}$$

【別解】 公式 $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ を使って展開すると、(1)、(2)は次のようになる。

$$(1) \quad (a-b+1)^2 = a^2 + (-b)^2 + 1^2 + 2 \times a \times (-b) + 2 \times (-b) \times 1 + 2 \times 1 \times a \\ = a^2 - 2ab + b^2 + 2a - 2b + 1$$

$$\begin{aligned}(2) \quad (2a-b-c)^2 &= (2a)^2 + (-b)^2 + (-c)^2 + 2 \times 2a \times (-b) + 2 \times (-b) \times (-c) \\ &\quad + 2 \times (-c) \times 2a \\ &= 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ca\end{aligned}$$

(3)～(6)も同様に公式を使って展開できる。

【11】

$$\text{【解答】} \quad (1) \quad x^2 + 2xy + y^2 + 5x + 5y + 6 \quad (2) \quad a^2 + 2ab + b^2 + 2a + 2b - 35$$

$$(3) \quad x^2 - y^2 + z^2 - 2zx \quad (4) \quad 4a^2 - 9b^2 - c^2 + 6bc$$

$$(5) \quad x^4 - 8x^3 + 13x^2 + 12x - 54 \quad (6) \quad x^4 - 10x^3y + 25x^2y^2 - 9y^4$$

$$(7) \quad -4x^4 + 12x^3 - 9x^2 + 16 \quad (8) \quad a^2 - b^2 + c^2 - d^2 - 2ac + 2bd$$

【解説】

$$\begin{aligned}(1) \quad (x+y+2)(x+y+3) &= ((x+y)+2)((x+y)+3) \\ &= (x+y)^2 + 5(x+y) + 6 \\ &= x^2 + 2xy + y^2 + 5x + 5y + 6\end{aligned}$$

$$\begin{aligned}(2) \quad (a+b-5)(a+b+7) &= ((a+b)-5)((a+b)+7) \\ &= (a+b)^2 + 2(a+b) - 35 \\ &= a^2 + 2ab + b^2 + 2a + 2b - 35\end{aligned}$$

$$\begin{aligned}(3) \quad (x+y-z)(x-y-z) &= ((x-z)+y)((x-z)-y) \\ &= (x-z)^2 - y^2 \\ &= x^2 - 2xz + z^2 - y^2 \\ &= x^2 - y^2 + z^2 - 2zx\end{aligned}$$

$$\begin{aligned}(4) \quad (2a-3b+c)(2a+3b-c) &= (2a-(3b-c))(2a+(3b-c)) \\ &= (2a)^2 - (3b-c)^2 \\ &= 4a^2 - (9b^2 - 6bc + c^2) \\ &= 4a^2 - 9b^2 + 6bc - c^2 \\ &= 4a^2 - 9b^2 - c^2 + 6bc\end{aligned}$$

$$\begin{aligned}(5) \quad (x^2-4x+6)(x^2-4x-9) &= ((x^2-4x)+6)((x^2-4x)-9) \\ &= (x^2-4x)^2 - 3(x^2-4x) - 54\end{aligned}$$

$$\begin{aligned}&= x^4 - 8x^3 + 16x^2 - 3x^2 + 12x - 54 \\ &= x^4 - 8x^3 + 13x^2 + 12x - 54\end{aligned}$$

$$\begin{aligned}(6) \quad (x^2-5xy+3y^2)(x^2-5xy-3y^2) &= ((x^2-5xy)+3y^2)((x^2-5xy)-3y^2) \\ &= (x^2-5xy)^2 - (3y^2)^2 \\ &= x^4 - 10x^3y + 25x^2y^2 - 9y^4\end{aligned}$$

$$\begin{aligned}(7) \quad (2x^2-3x+4)(4+3x-2x^2) &= [4+(2x^2-3x)][4-(2x^2-3x)] \\ &= 4^2 - (2x^2-3x)^2 \\ &= 16 - (4x^4 - 12x^3 + 9x^2) \\ &= -4x^4 + 12x^3 - 9x^2 + 16\end{aligned}$$

$$\begin{aligned}(8) \quad (a+b-c-d)(a-b-c+d) &= [(a-c)+(b-d)][(a-c)-(b-d)] \\ &= (a-c)^2 - (b-d)^2 \\ &= a^2 - 2ac + c^2 - (b^2 - 2bd + d^2) \\ &= a^2 - b^2 + c^2 - d^2 - 2ac + 2bd\end{aligned}$$

【12】

$$\text{【解答】} \quad (1) \quad 256a^4 - 288a^2b^2 + 81b^4 \quad (2) \quad 16x^4 - y^4 \quad (3) \quad x^4 - 13x^2 + 36 \\ (4) \quad x^4 + 4x^3 - 19x^2 - 46x + 120$$

【解説】

$$\begin{aligned}(1) \quad (4a+3b)^2(4a-3b)^2 &= [(4a+3b)(4a-3b)]^2 \\ &= (16a^2 - 9b^2)^2 \\ &= 256a^4 - 288a^2b^2 + 81b^4\end{aligned}$$

$$\begin{aligned}(2) \quad (4x^2+y^2)(2x+y)(2x-y) &= (4x^2+y^2)((2x)^2-y^2) \\ &= (4x^2+y^2)(4x^2-y^2) \\ &= (4x^2)^2 - (y^2)^2 \\ &= 16x^4 - y^4\end{aligned}$$

$$\begin{aligned}(3) \quad (x-2)(x+3)(x+2)(x-3) &= (x-2)(x+2) \times (x+3)(x-3) \\ &= (x^2-4)(x^2-9) \\ &= x^4 - 13x^2 + 36\end{aligned}$$

$$\begin{aligned}(4) \quad (x-2)(x-3)(x+4)(x+5) &= (x-2)(x+4) \times (x-3)(x+5) \\ &= (x^2+2x-8)(x^2+2x-15) \\ &= [(x^2+2x)-8][(x^2+2x)-15] \\ &= (x^2+2x)^2 - 23(x^2+2x) + 120 \\ &= x^4 + 4x^3 + 4x^2 - 23x^2 - 46x + 120 \\ &= x^4 + 4x^3 - 19x^2 - 46x + 120\end{aligned}$$

【13】

$$\text{【解答】} \quad (1) \quad 5 \quad (2) \quad -7 \quad (3) \quad 8$$

【解説】

$$(1) \quad \text{与えられた式を展開したとき、} x^3 \text{の項は} \\ (-2x) \times (-x^2) + 3x^2 \times x = 2x^3 + 3x^3 = 5x^3$$

よって、 x^3 の係数は 5

$$\begin{aligned}(2) \quad \text{与えられた式を展開したとき、} ab^2 \text{の項は} \\ 3ab \times (-3b) + b^2 \times 2a = -9ab^2 + 2ab^2 \\ = -7ab^2\end{aligned}$$

よって、 ab^2 の係数は -7

$$(1-2x+3x^2)(2+x-x^2)$$

$$(a^2+3ab+b^2)(2a-3b)$$

$$\begin{aligned}(3) \quad \text{与えられた式を展開したとき、} x^2 \text{の項は} \\ (-2x^2) \times 3 + 3x \times 4x + 2 \times x^2 \\ = -6x^2 + 12x^2 + 2x^2 = 8x^2\end{aligned}$$

よって、 x^2 の係数は 8

【14】

$$\text{【解答】} \quad (1) \quad x(x+3) \quad (2) \quad ab(b+2) \quad (3) \quad 3ab(b-9) \quad (4) \quad a(x+y-z)$$

$$(5) \quad x(3x^2-x+5) \quad (6) \quad (a+b)(x+y) \quad (7) \quad (x-y)(5a+2b)$$

$$(8) \quad \frac{1}{2}mn(m-3n) \quad (9) \quad \frac{x}{6}(18x^2-6x+1)$$

【解説】

$$(1) \quad x^2+3x = x \times x + x \times 3 = x(x+3)$$

$$(2) \quad ab^2+2ab = ab \times b + ab \times 2 = ab(b+2)$$

$$(3) \quad 3ab^2-27ab = 3ab \times b - 3ab \times 9 = 3ab(b-9)$$

$$(4) \quad ax+ay-az = a(x+y-z)$$

$$(5) \quad 3x^3-x^2+5x = x \times 3x^2 - x \times x + x \times 5 = x(3x^2-x+5)$$

$$(6) \quad x(a+b)+y(a+b) = (a+b)(x+y)$$

$$(7) \quad 5a(x-y)-2b(y-x) = 5a(x-y) + 2b(x-y) = (x-y)(5a+2b)$$

$$(8) \quad \frac{1}{2}m^2n - \frac{3}{2}mn^2 = \frac{1}{2}mn \times m - \frac{1}{2}mn \times n = \frac{1}{2}mn(m-3n)$$

$$(9) \quad 3x^3-x^2+\frac{1}{6}x = \frac{x}{6} \times 18x^2 - \frac{x}{6} \times 6x + \frac{x}{6} \times 1 = \frac{x}{6}(18x^2-6x+1)$$

【15】

$$\text{【解答】} \quad (1) \quad (x+1)(x+3) \quad (2) \quad (a+2)(a+4) \quad (3) \quad (x+5)(x-2)$$

$$(4) \quad (y-4)(y+7) \quad (5) \quad (x-1)(x-9) \quad (6) \quad (x+6y)(x+8y)$$

$$(7) \quad (x+3y)(x-12y) \quad (8) \quad (a-4b)(a-12b) \quad (9) \quad (p+7q)(p-5q)$$

【解説】

$$(1) \quad x^2+4x+3 = (x+1)(x+3)$$

$$(2) \quad a^2+6a+8 = (a+2)(a+4)$$

$$(3) \quad x^2+3x-10 = (x+5)(x-2)$$

$$(4) \quad y^2+3y-28 = (y-4)(y+7)$$

$$(5) \quad x^2-10x+9 = (x-1)(x-9)$$

$$(6) \quad x^2+14xy+48y^2 = (x+6y)(x+8y)$$

$$(7) \quad x^2-9xy-36y^2 = (x+3y)(x-12y)$$

$$(8) \quad a^2-16ab+48b^2 = (a-4b)(a-12b)$$

$$(9) \quad p^2+2pq-35q^2 = (p+7q)(p-5q)$$

【16】

$$\text{【解答】} \quad (1) \quad (x+2)^2 \quad (2) \quad (x-4)^2 \quad (3) \quad (6x+5)^2 \quad (4) \quad (9a-1)^2$$

$$(5) \quad (4x+7y)^2 \quad (6) \quad (a-2)^2 \quad (7) \quad (8m-1)^2 \quad (8) \quad (5p+11)^2$$

$$(9) \quad (13x-2y)^2$$

【解説】

$$(1) \quad x^2+4x+4 = x^2+2 \times 2 \times x+2^2 = (x+2)^2$$

$$(2) \quad x^2-8x+16 = x^2-2 \times 4 \times x+4^2 = (x-4)^2$$

$$(3) \quad 36x^2+60x+25 = (6x)^2+2 \times 6x \times 5+5^2 = (6x+5)^2$$

$$(4) \quad 81a^2-18a+1 = (9a)^2-2 \times 9a \times 1+1^2 = (9a-1)^2$$

中1 甲陽カンパリート数学 授業問題No6 ～展開・因数分解・平方根～【解答&解説】

$$(5) \quad 16x^2 + 56xy + 49y^2 = (4x)^2 + 2 \times 4x \times 7y + (7y)^2 = (4x + 7y)^2$$

$$(6) \quad 4 + a^2 - 4a = a^2 - 4a + 4 = a^2 - 2 \times 2 \times a + 2^2 = (a - 2)^2$$

$$(7) \quad 1 - 16m + 64m^2 = 64m^2 - 16m + 1 = (8m)^2 - 2 \times 8m \times \frac{1}{8} + \left(\frac{1}{8}\right)^2 = \left(8m - \frac{1}{8}\right)^2$$

$$(8) \quad 25p^2 + 110p + 121 = (5p)^2 + 2 \times 5p \times 11 + 11^2 = (5p + 11)^2$$

$$(9) \quad 169x^2 - 52xy + 4y^2 = (13x)^2 - 2 \times 13x \times 2y + (2y)^2 = (13x - 2y)^2$$

【17】

$$\text{【解答】} \quad (1) \quad (x+9)(x-9) \quad (2) \quad (3a+10)(3a-10) \quad (3) \quad (9p+7q)(9p-7q)$$

$$(4) \quad \left(\frac{a}{5} + \frac{b}{6}\right)\left(\frac{a}{5} - \frac{b}{6}\right) \quad (5) \quad (8t+11)(8t-11) \quad (6) \quad (14a+15b)(14a-15b)$$

【解説】

$$(1) \quad x^2 - 81 = x^2 - 9^2 = (x+9)(x-9)$$

$$(2) \quad 9a^2 - 100 = (3a)^2 - 10^2 = (3a+10)(3a-10)$$

$$(3) \quad -49q^2 + 81p^2 = 81p^2 - 49q^2 = (9p)^2 - (7q)^2 = (9p+7q)(9p-7q)$$

$$(4) \quad \frac{a^2}{25} - \frac{b^2}{36} = \left(\frac{a}{5}\right)^2 - \left(\frac{b}{6}\right)^2 = \left(\frac{a}{5} + \frac{b}{6}\right)\left(\frac{a}{5} - \frac{b}{6}\right)$$

$$(5) \quad 64t^2 - 121 = (8t)^2 - 11^2 = (8t+11)(8t-11)$$

$$(6) \quad 196a^2 - 225b^2 = (14a)^2 - (15b)^2 = (14a+15b)(14a-15b)$$

【18】

$$\text{【解答】} \quad (1) \quad 3a(x+2)(x+5) \quad (2) \quad 4t(x-2)(x-3) \quad (3) \quad -2m(x-3)(x+5)$$

$$(4) \quad 4p(x+3)(x-3) \quad (5) \quad \frac{1}{5}a(x-4)(x-7) \quad (6) \quad \frac{1}{7}t(a-2b)(a+5b)$$

$$(7) \quad \frac{1}{7}x(3a+4b)(3a-4b) \quad (8) \quad \frac{1}{5}k(7x-3y)^2$$

【解説】

$$(1) \quad 3ax^2 + 21ax + 30a = 3a(x^2 + 7x + 10) = 3a(x+2)(x+5)$$

$$(2) \quad 4tx^2 - 20tx + 24t = 4t(x^2 - 5x + 6) = 4t(x-2)(x-3)$$

$$(3) \quad -2mx^2 - 4mx + 30m = -2m(x^2 + 2x - 15) = -2m(x-3)(x+5)$$

$$(4) \quad 4px^2 - 36p = 4p(x^2 - 9) = 4p(x+3)(x-3)$$

$$(5) \quad \frac{1}{5}ax^2 - \frac{11}{5}ax + \frac{28}{5}a = \frac{1}{5}a(x^2 - 11x + 28) = \frac{1}{5}a(x-4)(x-7)$$

$$(6) \quad \frac{1}{7}a^2t + \frac{3}{7}abt - \frac{10}{7}b^2t = \frac{1}{7}t(a^2 + 3ab - 10b^2) = \frac{1}{7}t(a-2b)(a+5b)$$

$$(7) \quad \frac{9a^2x}{7} - \frac{16b^2x}{7} = \frac{1}{7}x(9a^2 - 16b^2) = \frac{1}{7}x(3a+4b)(3a-4b)$$

$$(8) \quad \frac{49}{5}kx^2 - \frac{42}{5}kxy + \frac{9}{5}ky^2 = \frac{1}{5}k(49x^2 - 42xy + 9y^2) = \frac{1}{5}k(7x-3y)^2$$

【19】

$$\text{【解答】} \quad (1) \quad b(a+b+c) \quad (2) \quad 2x(3ax+y+z) \quad (3) \quad (a+b)(x-y)$$

$$(4) \quad a(x+y)(a-x-y) \quad (5) \quad (a-4)(5a+2) \quad (6) \quad (x-3y)(2x+y)$$

【解説】

$$(1) \quad b^2 + b(a+c) = b[b + (a+c)] \\ = b(a+b+c)$$

$$(2) \quad 2x(y+z) + 6ax^2 = 2x[(y+z) + 3ax] \\ = 2x(3ax+y+z)$$

$$(3) \quad x(a+b) - y(a+b) = (a+b)(x-y)$$

$$(4) \quad a^2(x+y) - a(x+y)^2 = a(x+y)[a - (x+y)]$$

$$= a(x+y)(a-x-y)$$

$$(5) \quad 5a(a-4) - 2(4-a) = 5a(a-4) - 2[-(a-4)]$$

$$= 5a(a-4) + 2(a-4)$$

$$= (a-4)(5a+2)$$

$$(6) \quad 2x(x-3y) - y(3y-x) = 2x(x-3y) - y[-(x-3y)]$$

$$= 2x(x-3y) + y(x-3y)$$

$$= (x-3y)(2x+y)$$

【20】

$$\text{【解答】} \quad (1) \quad (x+2)(x-4) \quad (2) \quad (a-2)(a-8) \quad (3) \quad (x+2y)(6x-y)$$

$$(4) \quad (2a-3b)(5a-b)$$

【解説】

$$(1) \quad (x+5)(x-7) + 27 = (x^2 - 2x - 35) + 27$$

$$= x^2 - 2x - 8$$

$$= (x+2)(x-4)$$

$$(2) \quad (a-4)(a-6) - 8 = (a^2 - 10a + 24) - 8$$

$$= a^2 - 10a + 16$$

$$= (a-2)(a-8)$$

$$(3) \quad (2x-y)(3x+2y) + 10xy = (6x^2 + xy - 2y^2) + 10xy$$

$$= 6x^2 + 11xy - 2y^2$$

$$= (x+2y)(6x-y)$$

$$(4) \quad (2a-b)(5a-6b) - 3b^2 = (10a^2 - 17ab + 6b^2) - 3b^2$$

$$= 10a^2 - 17ab + 3b^2$$

$$= (2a-3b)(5a-b)$$

【21】

$$\text{【解答】} \quad (1) \quad (x-2)(3x+1) \quad (2) \quad (2x+3)(3x+2) \quad (3) \quad (a+3)(4a-1)$$

$$(4) \quad (x-7y)(3x-5y) \quad (5) \quad (2x+3y)(2x+5y) \quad (6) \quad (3x+y)(3x-2y)$$

【解説】

$$(1) \quad 3x^2 - 5x - 2$$

$$= (x-2)(3x+1)$$

$$(2) \quad 6x^2 + 13x + 6$$

$$= (2x+3)(3x+2)$$

$$\begin{array}{ccc} 1 & \times & -2 \longrightarrow -6 \\ 3 & & 1 \longrightarrow 1 \\ \hline & 3 & -2 \longrightarrow -5 \end{array}$$

$$\begin{array}{ccc} 2 & \times & 3 \longrightarrow 9 \\ 3 & & 2 \longrightarrow 4 \\ \hline & 6 & 6 \longrightarrow 13 \end{array}$$

$$(3) \quad 4a^2 + 11a - 3$$

$$= (a+3)(4a-1)$$

$$(4) \quad 3x^2 - 26xy + 35y^2$$

$$= (x-7y)(3x-5y)$$

$$\begin{array}{ccc} 1 & \times & 3 \longrightarrow 12 \\ 4 & & -1 \longrightarrow -1 \\ \hline & 4 & -3 \longrightarrow 11 \end{array}$$

$$\begin{array}{ccc} 1 & \times & -7 \longrightarrow -21 \\ 3 & & -5 \longrightarrow -5 \\ \hline & 3 & 35 \longrightarrow -26 \end{array}$$

$$(5) \quad 4x^2 + 16xy + 15y^2$$

$$= (2x+3y)(2x+5y)$$

$$(6) \quad 9x^2 - 3xy - 2y^2$$

$$= (3x+y)(3x-2y)$$

$$\begin{array}{ccc} 2 & \times & 3 \longrightarrow 6 \\ 2 & & 5 \longrightarrow 10 \\ \hline & 4 & 15 \longrightarrow 16 \end{array}$$

$$\begin{array}{ccc} 3 & \times & 1 \longrightarrow 3 \\ 3 & & -2 \longrightarrow -6 \\ \hline & 9 & -2 \longrightarrow -3 \end{array}$$

【22】

$$\text{【解答】} \quad (1) \quad (2x+1)(2y+1) \quad (2) \quad (x+5)(x-1) \quad (3) \quad (x+2)(x-2)$$

$$(4) \quad (x^2 - 2x - 1)(x+1)(x-3) \quad (5) \quad (x-2)(x-4)(x+1)(x-7)$$

$$(6) \quad (x+2)^2(x+6)(x-2)$$

【解説】

$$(1) \quad (x+y+1)^2 - (x-y)^2 = [(x+y+1) + (x-y)][(x+y+1) - (x-y)] \\ = (2x+1)(2y+1)$$

$$(2) \quad (x+1)^2 + 2(x+1) - 8 = [(x+1) + 4][(x+1) - 2] \\ = (x+5)(x-1)$$

$$(3) \quad (x+1)^2 - 2(x+1) - 3 = [(x+1) + 1][(x+1) - 3] \\ = (x+2)(x-2)$$

$$(4) \quad (x^2 - 2x)^2 - 4(x^2 - 2x) + 3 = [(x^2 - 2x) - 1][(x^2 - 2x) - 3] \\ = (x^2 - 2x - 1)(x^2 - 2x - 3) \\ = (x^2 - 2x - 1)(x+1)(x-3)$$

$$(5) \quad (x^2 - 6x)^2 + (x^2 - 6x) - 56 = [(x^2 - 6x) + 8][(x^2 - 6x) - 7] \\ = (x^2 - 6x + 8)(x^2 - 6x - 7) \\ = (x-2)(x-4)(x+1)(x-7)$$

$$(6) \quad (x^2 + 4x)^2 - 8(x^2 + 4x) - 48 = [(x^2 + 4x) + 4][(x^2 + 4x) - 12] \\ = (x^2 + 4x + 4)(x^2 + 4x - 12) \\ = (x+2)^2(x+6)(x-2)$$

【23】

$$\text{【解答】} \quad (1) \quad (a+b-3)(c+2) \quad (2) \quad (x-y)(x+y-1) \quad (3) \quad (x-y)(-xy+1)$$

$$(4) \quad (x+y+1)(-x-y+1) \quad (5) \quad (x+1)(x-1)^2 \quad (6) \quad (a-b)(a-b-6)$$

【解説】

$$(1) \quad ac + bc + 2a + 2b - 3c - 6 = (ac + bc - 3c) + (2a + 2b - 6) \\ = c(a+b-3) + 2(a+b-3) \\ = (a+b-3)(c+2)$$

$$(2) \quad x^2 - y^2 - x + y = (x^2 - y^2) - (x-y) = (x+y)(x-y) - (x-y) \\ = (x-y)(x+y-1)$$

$$(3) \quad xy^2 - y - x^2y + x = (xy^2 - x^2y) + (x-y) \\ = x y(y-x) + (x-y) \\ = -x y(x-y) + (x-y) \\ = (x-y)(-xy+1)$$

$$(4) \quad 1 - x^2 - 2xy - y^2 = 1 - (x^2 + 2xy + y^2) \\ = 1 - (x+y)^2 \\ = [1 + (x+y)][1 - (x+y)] \\ = (x+y+1)(-x-y+1)$$

$$(5) \quad x^3 - x^2 - x + 1 = (x^3 - x^2) - (x-1) \\ = x^2(x-1) - (x-1) = (x-1)(x^2-1) \\ = (x-1)(x+1)(x-1) = (x+1)(x-1)^2$$

$$(6) \quad a^2 - 2ab + b^2 - 6a + 6b = (a^2 - 2ab + b^2) - (6a - 6b) \\ = (a-b)^2 - 6(a-b) \\ = (a-b)[(a-b) - 6] \\ = (a-b)(a-b-6)$$

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24

- 【解答】 (1) $2(x+1)(x-1)(x+2)(x-2)$ (2) $(a-b)(x+2)(x-2)$
(3) $(x+2y-2)(x-2y+2)$ (4) $(2x+3y-z)(2x-3y-z)$

【解説】

$$(1) \quad 2(x^2-1)^2-6x^2+6=2(x^2-1)^2-6(x^2-1) \\ =2(x^2-1)\{(x^2-1)-3\} \\ =2(x^2-1)(x^2-4) \\ =2(x+1)(x-1)(x+2)(x-2)$$

$$(2) \quad (a-b)x^2-4a+4b=(a-b)x^2-4(a-b) \\ =(a-b)(x^2-4) \\ =(a-b)(x+2)(x-2)$$

$$(3) \quad x^2-4y^2+8y-4=x^2-4(y^2-2y+1) \\ =x^2-4(y-1)^2 \\ =x^2-[2(y-1)]^2 \\ =[x+2(y-1)][x-2(y-1)] \\ =(x+2y-2)(x-2y+2)$$

$$(4) \quad 4x^2-9y^2+z^2-4xz=(4x^2-4xz+z^2)-9y^2 \\ =(2x-z)^2-(3y)^2 \\ =[(2x-z)+3y][(2x-z)-3y] \\ =(2x+3y-z)(2x-3y-z)$$

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- 【解答】 (1) $(x+1)(x-1)(x+2)(x-2)$ (2) $(x^2+4)(x^2-3)$
(3) $(x+2)(x-2)(x+4)(x-4)$ (4) $(x^2+3)(x+4)(x-4)$
(5) $(x+y)(x-y)(x+3y)(x-3y)$ (6) $(x^2+x+2)(x^2-x+2)$
(7) $(x^2+x+3)(x^2-x+3)$ (8) $(x^2+4xy-y^2)(x^2-4xy-y^2)$
(9) $(2x^2+xy+3y^2)(2x^2-xy+3y^2)$

【解説】

$$(1) \quad x^4-5x^2+4=(x^2)^2-5x^2+4=(x^2-1)(x^2-4) \\ =(x+1)(x-1)(x+2)(x-2)$$

$$(2) \quad x^4+x^2-12=(x^2)^2+x^2-12=(x^2+4)(x^2-3)$$

$$(3) \quad x^4-20x^2+64=(x^2)^2-20x^2+64=(x^2-4)(x^2-16) \\ =(x+2)(x-2)(x+4)(x-4)$$

$$(4) \quad x^4-13x^2-48=(x^2)^2-13x^2-48=(x^2+3)(x^2-16) \\ =(x^2+3)(x+4)(x-4)$$

$$(5) \quad x^4-10x^2y^2+9y^4=(x^2)^2-10x^2y^2+9(y^2)^2 \\ =(x^2-y^2)(x^2-9y^2) \\ =(x+y)(x-y)(x+3y)(x-3y)$$

$$(6) \quad x^4+3x^2+4=(x^4+4x^2+4)-x^2=(x^2+2)^2-x^2 \\ =[(x^2+2)+x][(x^2+2)-x] \\ =(x^2+x+2)(x^2-x+2)$$

$$(7) \quad x^4+5x^2+9=(x^4+6x^2+9)-x^2=(x^2+3)^2-x^2 \\ =[(x^2+3)+x][(x^2+3)-x] \\ =(x^2+x+3)(x^2-x+3)$$

$$(8) \quad x^4-18x^2y^2+y^4=(x^4-2x^2y^2+y^4)-16x^2y^2 \\ =(x^2-y^2)^2-(4xy)^2 \\ =[(x^2-y^2)+4xy][(x^2-y^2)-4xy] \\ =(x^2+4xy-y^2)(x^2-4xy-y^2)$$

$$(9) \quad 4x^4+11x^2y^2+9y^4=(4x^4+12x^2y^2+9y^4)-x^2y^2 \\ =(2x^2+3y^2)^2-(xy)^2 \\ =[(2x^2+3y^2)+xy][(2x^2+3y^2)-xy] \\ =(2x^2+xy+3y^2)(2x^2-xy+3y^2)$$

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- 【解答】 (1) 63.8401 (2) 1599 (3) 8.9996 (4) 10506 (5) 31.4 (6) 22500

【解説】

$$(1) \quad 7.99^2=(8-0.01)^2=8^2-2\times 8\times 0.01+0.01^2 \\ =64-0.16+0.0001=63.8401$$

$$(2) \quad 39\times 41=(40-1)\times (40+1)=40^2-1^2 \\ =1600-1=1599$$

$$(3) \quad 3.02\times 2.98=(3+0.02)\times (3-0.02)=3^2-0.02^2 \\ =9-0.0004=8.9996$$

$$(4) \quad 102\times 103=(100+2)\times (100+3)=100^2+5\times 100+6 \\ =10000+500+6=10506$$

$$(5) \quad 5.5^2\times 3.14-4.5^2\times 3.14=(5.5^2-4.5^2)\times 3.14 \\ =(5.5+4.5)\times (5.5-4.5)\times 3.14 \\ =10\times 1\times 3.14=31.4$$

$$(6) \quad 153^2+147^2+156\times 144-2\times 153\times 147=(153^2-2\times 153\times 147+147^2)+156\times 144 \\ =(153-147)^2+(150+6)\times (150-6) \\ =6^2+150^2-6^2=150^2=22500$$

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- 【解答】 (1) -39 (2) 10

【解説】

$$(1) \quad (2a+b)^2-4a(a-b)=4a^2+4ab+b^2-4a^2+4ab \\ =8ab+b^2=8\times 2\times (-3)+(-3)^2 \\ =-48+9=-39$$

$$(2) \quad a^2+4b^2-4ab+1=(a-2b)^2+1=(3.76-2\times 0.38)^2+1 \\ =(3.76-0.76)^2+1 \\ =3^2+1=10$$

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- 【解答】 (1) -21 (2) 21

【解説】

$$(1) \quad a^2+a^2b+2ab+ab^2+b^2=(a^2+2ab+b^2)+a^2b+ab^2 \\ =(a+b)^2+ab(a+b) \\ =3^2+(-10)\times 3 \\ =9-30=-21$$

$$(2) \quad a^2+b^2=(a-b)^2+2ab=(-5)^2+2\times (-2) \\ =25-4=21$$

29

- 【解答】 (1) 6 と -6 (±6) (2) 100 と -100 (±100) (3) 11 と -11 (±11)
(4) 70 と -70 (±70) (5) 0.3 と -0.3 (±0.3) (6) 0.4 と -0.4 (±0.4)
(7) $\frac{6}{13}$ と $-\frac{6}{13}$ (± $\frac{6}{13}$) (8) $\frac{12}{19}$ と $-\frac{12}{19}$ (± $\frac{12}{19}$)

【解説】

- (1) 6 と -6 (2) 100 と -100 (3) 11 と -11
(4) 70 と -70 (5) 0.3 と -0.3 (6) 0.4 と -0.4
(7) $\frac{6}{13}$ と $-\frac{6}{13}$ (8) $\frac{12}{19}$ と $-\frac{12}{19}$

上の答えは、複号 ± を使って次のように書いてもよい。

- (1) ±6 (2) ±100 (3) ±11 (4) ±70 (5) ±0.3 (6) ±0.4
(7) ± $\frac{6}{13}$ (8) ± $\frac{12}{19}$

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- 【解答】 (1) ± $\sqrt{5}$ (2) ± $\sqrt{231}$ (3) ± $\sqrt{1.41}$ (4) ± $\sqrt{2\frac{1}{3}}$

【解説】

- (1) ± $\sqrt{5}$ (2) ± $\sqrt{231}$ (3) ± $\sqrt{1.41}$ (4) ± $\sqrt{2\frac{1}{3}}$

31

- 【解答】 (1) 1 (2) 9 (3) -5 (4) -7 (5) 18 (6) -9 (7) 6
(8) -9 (9) 0.7 (10) 30 (11) 0.14 (12) 70

【解説】

- (1) $\sqrt{1}=\sqrt{1^2}=1$ (2) $\sqrt{81}=\sqrt{9^2}=9$
(3) $-\sqrt{25}=-\sqrt{5^2}=-5$ (4) $-\sqrt{49}=-\sqrt{7^2}=-7$
(5) $\sqrt{18^2}=18$ (6) $-\sqrt{9^2}=-9$
(7) $\sqrt{(-6)^2}=\sqrt{6^2}=6$ (8) $-\sqrt{(-9)^2}=-\sqrt{9^2}=-9$
(9) $\sqrt{0.49}=\sqrt{0.7^2}=0.7$ (10) $\sqrt{900}=\sqrt{30^2}=30$
(11) $\sqrt{0.0196}=\sqrt{0.14^2}=0.14$ (12) $\sqrt{4900}=\sqrt{70^2}=70$

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- 【解答】 (1) $\frac{3}{2}$ (2) $-\frac{3}{8}$ (3) $\frac{4}{3}$ (4) $\frac{2}{3}$

【解説】

- (1) $\sqrt{\frac{81}{36}}=\sqrt{\frac{9}{4}}=\sqrt{\left(\frac{3}{2}\right)^2}=\frac{3}{2}$
(2) $-\sqrt{\frac{9}{64}}=-\sqrt{\left(\frac{3}{8}\right)^2}=-\frac{3}{8}$
(3) $\sqrt{\left(\frac{4}{3}\right)^2}=\frac{4}{3}$
(4) $\sqrt{\left(-\frac{2}{3}\right)^2}=\sqrt{\left(\frac{2}{3}\right)^2}=\frac{2}{3}$

33

- 【解答】 (1) 6 (2) 6 (3) -7 (4) -10 (5) 13 (6) $\frac{3}{4}$ (7) $\frac{1}{2}$

(8) $\frac{3}{2}$ (9) 121

【解説】

- (1) $(\sqrt{6})^2=6$ (2) $(-\sqrt{6})^2=6$
 (3) $-(\sqrt{7})^2=-7$ (4) $-(-\sqrt{10})^2=-10$
 (5) $\{-(\sqrt{13})\}^2=(\sqrt{13})^2=13$ (6) $(\sqrt{\frac{3}{4}})^2=\frac{3}{4}$
 (7) $(-\sqrt{\frac{1}{2}})^2=\frac{1}{2}$ (8) $(-\sqrt{\frac{3}{2}})^2=\frac{3}{2}$
 (9) $(-\sqrt{11})^4=[(-\sqrt{11})^2]^2=11^2=121$

【34】

【解答】 (1) $6 > \sqrt{35}$ (2) $-9 < -\sqrt{80}$ (3) $-\frac{1}{2} < \frac{1}{3} < \sqrt{\frac{1}{2}}$

【解説】

- (1) $6 = \sqrt{6^2} = \sqrt{36}$, $36 > 35$ であるから
 $\sqrt{36} > \sqrt{35}$ すなわち $6 > \sqrt{35}$
 (2) $9 = \sqrt{9^2} = \sqrt{81}$, $81 > 80$ であるから
 $\sqrt{81} > \sqrt{80}$ すなわち $9 > \sqrt{80}$
 よって $-9 < -\sqrt{80}$

- (3) $-\frac{1}{2}$ は負の数, $\sqrt{\frac{1}{2}}$, $\frac{1}{3}$ は正の数であるから, 3つの数の中で $-\frac{1}{2}$ が最も小さい。

$\frac{1}{3} = \sqrt{(\frac{1}{3})^2} = \sqrt{\frac{1}{9}}$, $\frac{1}{9} < \frac{1}{2}$ であるから

$\sqrt{\frac{1}{9}} < \sqrt{\frac{1}{2}}$ すなわち $\frac{1}{3} < \sqrt{\frac{1}{2}}$

よって $-\frac{1}{2} < \frac{1}{3} < \sqrt{\frac{1}{2}}$

【別解】 (1) $6^2=36$, $(\sqrt{35})^2=35$ であるから $6^2 > (\sqrt{35})^2$

よって $6 > \sqrt{35}$

(2) $9^2=81$, $(\sqrt{80})^2=80$ であるから $9^2 > (\sqrt{80})^2$

よって $9 > \sqrt{80}$

したがって $-9 < -\sqrt{80}$

(3) $(\frac{1}{3})^2 = \frac{1}{9}$, $(\sqrt{\frac{1}{2}})^2 = \frac{1}{2}$, $\frac{1}{9} < \frac{1}{2}$ であるから $(\frac{1}{3})^2 < (\sqrt{\frac{1}{2}})^2$

よって $\frac{1}{3} < \sqrt{\frac{1}{2}}$

したがって $-\frac{1}{2} < \frac{1}{3} < \sqrt{\frac{1}{2}}$

【35】

- 【解答】 (1) 正しい (2) 正しくない (3) 正しくない (4) 正しい
 (5) 正しい (6) 正しくない (7) 正しい (8) 正しくない

【解説】

- (1) 正しい
 (2) 正しくない [-4 の平方は 16]
 (3) 正しくない [$\sqrt{7}$ の平方は 7]

- (4) 正しい
 (5) 正しい
 (6) 正しくない [81 の平方根は 9 と -9]
 (7) 正しい
 (8) 正しくない [$\sqrt{0}=0$ で, 平方根は存在する]

【36】

【解答】 (1) $\sqrt{30}$ (2) $\sqrt{42}$ (3) $\sqrt{\frac{2}{7}}$ (4) $\sqrt{\frac{1}{3}}$

【解説】

- (1) $\sqrt{6} \times \sqrt{5} = \sqrt{6 \times 5} = \sqrt{30}$
 (2) $\sqrt{3} \sqrt{14} = \sqrt{3 \times 14} = \sqrt{42}$
 (3) $\frac{\sqrt{2}}{\sqrt{7}} = \sqrt{\frac{2}{7}}$
 (4) $\sqrt{14} \div \sqrt{42} = \frac{\sqrt{14}}{\sqrt{42}} = \sqrt{\frac{14}{42}} = \sqrt{\frac{1}{3}}$

【37】

【解答】 (1) $\sqrt{75}$ (2) $\sqrt{\frac{7}{2}}$ (3) $\sqrt{\frac{9}{20}}$ (4) $\sqrt{5}$

【解説】

- (1) $5\sqrt{3} = \sqrt{5^2 \times 3} = \sqrt{5^2 \times 3} = \sqrt{75}$
 (2) $\frac{\sqrt{14}}{2} = \frac{\sqrt{14}}{\sqrt{2^2}} = \sqrt{\frac{14}{2^2}} = \sqrt{\frac{7}{2}}$
 (3) $\frac{3\sqrt{5}}{10} = \frac{\sqrt{3^2 \times 5}}{\sqrt{10^2}} = \sqrt{\frac{3^2 \times 5}{10^2}} = \sqrt{\frac{9}{20}}$
 (4) $\frac{\sqrt{8} \sqrt{15}}{2\sqrt{6}} = \frac{\sqrt{8 \times 15}}{\sqrt{2^2 \times 6}} = \frac{\sqrt{8 \times 15}}{\sqrt{2^2 \times 6}} = \sqrt{5}$

【38】

【解答】 (1) $2\sqrt{10}$ (2) $11\sqrt{2}$ (3) $4\sqrt{6}$ (4) $50\sqrt{2}$ (5) $\frac{1}{3}\sqrt{5}$
 (6) $\frac{1}{6}\sqrt{11}$ (7) $\frac{1}{5}\sqrt{7}$ (8) $\frac{1}{20}\sqrt{5}$

【解説】

- (1) $\sqrt{40} = \sqrt{2^2 \times 10} = \sqrt{2^2} \sqrt{10} = 2\sqrt{10}$
 (2) $\sqrt{242} = \sqrt{11^2 \times 2} = \sqrt{11^2} \sqrt{2} = 11\sqrt{2}$
 (3) $\sqrt{96} = \sqrt{4^2 \times 6} = \sqrt{4^2} \sqrt{6} = 4\sqrt{6}$
 (4) $\sqrt{5000} = \sqrt{50^2 \times 2} = \sqrt{50^2} \sqrt{2} = 50\sqrt{2}$
 (5) $\sqrt{\frac{5}{9}} = \sqrt{\frac{5}{3^2}} = \frac{\sqrt{5}}{\sqrt{3^2}} = \frac{\sqrt{5}}{3} = \frac{1}{3}\sqrt{5}$
 (6) $\sqrt{\frac{11}{36}} = \sqrt{\frac{11}{6^2}} = \frac{\sqrt{11}}{\sqrt{6^2}} = \frac{\sqrt{11}}{6} = \frac{1}{6}\sqrt{11}$
 (7) $\sqrt{0.28} = \sqrt{\frac{28}{100}} = \sqrt{\frac{7}{25}} = \sqrt{\frac{7}{5^2}} = \frac{\sqrt{7}}{\sqrt{5^2}} = \frac{\sqrt{7}}{5} = \frac{1}{5}\sqrt{7}$
 (8) $\sqrt{0.0125} = \sqrt{\frac{125}{10000}} = \sqrt{\frac{5^2 \times 5}{100^2}} = \sqrt{\frac{5}{20^2}} = \frac{\sqrt{5}}{\sqrt{20^2}} = \frac{\sqrt{5}}{20} = \frac{1}{20}\sqrt{5}$

【39】

【解答】 (1) 9 (2) $6\sqrt{2}$ (3) 2 (4) 2 (5) $\frac{\sqrt{14}}{6}$

【解説】

- (1) $\sqrt{3} \times \sqrt{27} = \sqrt{3 \times 27} = \sqrt{81} = \sqrt{9^2} = 9$
 (2) $\sqrt{3} \times \sqrt{27} = \sqrt{3} \times \sqrt{3^2 \times 3} = \sqrt{3} \times 3\sqrt{3} = 3 \times (\sqrt{3})^2 = 9$
 (3) $\sqrt{12} \div \sqrt{3} = \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$

【別解】 $\sqrt{12} \div \sqrt{3} = \sqrt{2^2 \times 3} \div \sqrt{3} = 2\sqrt{3} \div \sqrt{3} = 2$

(4) $\sqrt{2} \div \sqrt{3} \times \sqrt{6} = \frac{\sqrt{2} \times \sqrt{6}}{\sqrt{3}} = \sqrt{\frac{2 \times 6}{3}} = \sqrt{4} = 2$

(5) $3\sqrt{5} \times \sqrt{\frac{7}{12}} \div 6\sqrt{\frac{15}{8}} = 3\sqrt{5} \times \sqrt{\frac{7}{12}} \times \frac{1}{6}\sqrt{\frac{8}{15}}$
 $= \frac{3}{6} \sqrt{\frac{5 \times 7 \times 8}{12 \times 15}} = \frac{1}{2} \sqrt{\frac{14}{9}}$
 $= \frac{1}{2} \times \frac{\sqrt{14}}{3} = \frac{\sqrt{14}}{6}$

【40】

【解答】 (1) $\frac{\sqrt{2}}{2}$ (2) $\frac{5\sqrt{3}}{3}$ (3) $\frac{5\sqrt{2}}{7}$ (4) $\frac{\sqrt{14}}{2}$ (5) $\frac{2\sqrt{6}}{9}$

【解説】

- (1) $\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$
 (2) $\frac{5}{\sqrt{3}} = \frac{5 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{5\sqrt{3}}{3}$
 (3) $\frac{10}{7\sqrt{2}} = \frac{10 \times \sqrt{2}}{7\sqrt{2} \times \sqrt{2}} = \frac{10\sqrt{2}}{7 \times 2} = \frac{5\sqrt{2}}{7}$
 (4) $\frac{\sqrt{21}}{\sqrt{6}} = \frac{\sqrt{21} \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{\sqrt{126}}{6} = \frac{3\sqrt{14}}{6} = \frac{\sqrt{14}}{2}$
 (5) $\frac{\sqrt{8}}{\sqrt{27}} = \frac{2\sqrt{2}}{3\sqrt{3}} = \frac{2\sqrt{2} \times \sqrt{3}}{3\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{6}}{9}$

【41】

【解答】 0.7938

【解説】

$\frac{21}{10\sqrt{7}} = \frac{21 \times \sqrt{7}}{10\sqrt{7} \times \sqrt{7}} = \frac{21\sqrt{7}}{10 \times 7} = \frac{3\sqrt{7}}{10}$
 $= \frac{3 \times 2.646}{10} = \frac{7.938}{10} = 0.7938$

【42】

【解答】 (1) $17\sqrt{7}$ (2) $\sqrt{2}$ (3) $\sqrt{2}$ (4) $3\sqrt{3}$ (5) 0 (6) $\frac{5\sqrt{3}}{7}$
 (7) $\sqrt{2}$ (8) $\sqrt{2}$ (9) 0 (10) $-\sqrt{5}$ (11) $\sqrt{2} - \sqrt{3}$

【解説】

- (1) $4\sqrt{7} + 13\sqrt{7} = (4+13)\sqrt{7} = 17\sqrt{7}$
 (2) $\sqrt{50} - \sqrt{32} = 5\sqrt{2} - 4\sqrt{2} = (5-4)\sqrt{2} = \sqrt{2}$

$$\begin{aligned}(3) \quad & \sqrt{32} - \sqrt{72} + 3\sqrt{2} = 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = \sqrt{2} \\(4) \quad & 2\sqrt{75} - \sqrt{48} - 3\sqrt{3} = 10\sqrt{3} - 4\sqrt{3} - 3\sqrt{3} = 3\sqrt{3} \\(5) \quad & 2\sqrt{5} + 3\sqrt{80} - \sqrt{20} - 2\sqrt{180} = 2\sqrt{5} + 12\sqrt{5} - 2\sqrt{5} - 12\sqrt{5} = 0 \\(6) \quad & \sqrt{\frac{3}{49}} + \frac{4\sqrt{3}}{7} = \frac{\sqrt{3}}{7} + \frac{4\sqrt{3}}{7} = \frac{5\sqrt{3}}{7} \\(7) \quad & \frac{1}{\sqrt{2}} + \sqrt{10} \times \frac{1}{2\sqrt{5}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}\end{aligned}$$

$$(8) \quad \sqrt{18} - \frac{\sqrt{24}}{\sqrt{3}} = 3\sqrt{2} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$

$$(9) \quad 4\sqrt{2} - \sqrt{50} + \frac{\sqrt{8}}{2} = 4\sqrt{2} - 5\sqrt{2} + \frac{2\sqrt{2}}{2} = 0$$

$$\begin{aligned}(10) \quad & 3\sqrt{20} - \frac{15}{\sqrt{5}} - \sqrt{80} = 3\sqrt{20} - \frac{15 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} - \sqrt{80} \\& = 6\sqrt{5} - 3\sqrt{5} - 4\sqrt{5} \\& = -\sqrt{5}\end{aligned}$$

$$\begin{aligned}(11) \quad & \frac{15\sqrt{2}}{\sqrt{6}} - \frac{4}{\sqrt{2}} - \frac{18}{\sqrt{3}} + \sqrt{18} = \frac{15}{\sqrt{3}} - \frac{4}{\sqrt{2}} - \frac{18}{\sqrt{3}} + 3\sqrt{2} \\& = \frac{15 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} - \frac{4 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} - \frac{18 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} + 3\sqrt{2} \\& = 5\sqrt{3} - 2\sqrt{2} - 6\sqrt{3} + 3\sqrt{2} \\& = \sqrt{2} - \sqrt{3}\end{aligned}$$

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解答 (1) $3\sqrt{2}$ (2) $\sqrt{2}$ (3) $5 - \sqrt{10} + \sqrt{15} - \sqrt{6}$ (4) $3 + 2\sqrt{2}$
(5) $13 - 4\sqrt{10}$ (6) $-5\sqrt{6}$ (7) $21 - 11\sqrt{6}$ (8) 5 (9) $3\sqrt{2}$
(10) 5 (11) $-8 - \sqrt{6}$

解説

$$(1) \quad \sqrt{3}(\sqrt{24} - \sqrt{6}) = \sqrt{3}(2\sqrt{6} - \sqrt{6}) = \sqrt{3} \cdot \sqrt{6} = 3\sqrt{2}$$

$$(2) \quad (\sqrt{2} + 2)(\sqrt{2} - 1) = (\sqrt{2})^2 + (2 - 1)\sqrt{2} + 2 \times (-1) = 2 + \sqrt{2} - 2 = \sqrt{2}$$

$$(3) \quad (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{3}) = (\sqrt{5})^2 + (-\sqrt{2} + \sqrt{3})\sqrt{5} - \sqrt{2}\sqrt{3} = 5 - \sqrt{10} + \sqrt{15} - \sqrt{6}$$

$$(4) \quad (\sqrt{2} + 1)^2 = (\sqrt{2})^2 + 2 \times \sqrt{2} \times 1 + 1^2 = 2 + 2\sqrt{2} + 1 = 3 + 2\sqrt{2}$$

$$\begin{aligned}(5) \quad & (\sqrt{8} - \sqrt{5})^2 = (2\sqrt{2} - \sqrt{5})^2 \\& = (2\sqrt{2})^2 - 2 \times 2\sqrt{2} \times \sqrt{5} + (\sqrt{5})^2 \\& = 8 - 4\sqrt{10} + 5 = 13 - 4\sqrt{10}\end{aligned}$$

$$(6) \quad (\sqrt{2} - 2\sqrt{3})(3\sqrt{2} + \sqrt{3}) = 3(\sqrt{2})^2 + \{1 \times \sqrt{3} + (-2\sqrt{3}) \times 3\}\sqrt{2} + (-2\sqrt{3}) \times \sqrt{3} = 6 - 5\sqrt{6} - 6 = -5\sqrt{6}$$

$$\begin{aligned}(7) \quad & (\sqrt{3} - 3\sqrt{2})(\sqrt{27} - \sqrt{8}) \\& = (\sqrt{3} - 3\sqrt{2})(3\sqrt{3} - 2\sqrt{2}) \\& = 3(\sqrt{3})^2 + \{1 \times (-2\sqrt{2}) + (-3\sqrt{2}) \times 3\}\sqrt{3} + (-3\sqrt{2}) \times (-2\sqrt{2}) \\& = 9 - 11\sqrt{6} + 12 \\& = 21 - 11\sqrt{6}\end{aligned}$$

$$(8) \quad (\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) = (\sqrt{7})^2 - (\sqrt{2})^2$$

$$= 7 - 2 = 5$$

$$(9) \quad \sqrt{6}(\sqrt{3} - \sqrt{2}) + 2\sqrt{3} = 3\sqrt{2} - 2\sqrt{3} + 2\sqrt{3} = 3\sqrt{2}$$

$$\begin{aligned}(10) \quad & (\sqrt{6} - \sqrt{2})^2 + \sqrt{3}(4 - \sqrt{3}) = (\sqrt{6})^2 - 2 \times \sqrt{6} \times \sqrt{2} + (\sqrt{2})^2 + 4\sqrt{3} - (\sqrt{3})^2 \\& = 6 - 4\sqrt{3} + 2 + 4\sqrt{3} - 3 = 5\end{aligned}$$

$$\begin{aligned}(11) \quad & \sqrt{3}(\sqrt{12} + \sqrt{18}) - (2\sqrt{3} + \sqrt{2})^2 \\& = \sqrt{3}(2\sqrt{3} + 3\sqrt{2}) - \{(2\sqrt{3})^2 + 2 \times 2\sqrt{3} \times \sqrt{2} + (\sqrt{2})^2\} \\& = 6 + 3\sqrt{6} - (12 + 4\sqrt{6} + 2) \\& = -8 - \sqrt{6}\end{aligned}$$

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解答 (1) $-2 - 10\sqrt{2}$ (2) $\sqrt{2}$ (3) 6 (4) $10 + 2\sqrt{6} - 2\sqrt{10} - 2\sqrt{15}$
(5) $4\sqrt{2} + 4\sqrt{6}$ (6) $\sqrt{5} + \sqrt{7}$

解説

$$\begin{aligned}(1) \quad & (2\sqrt{2} - 1)^2 - (\sqrt{2} + 3)^2 = (2\sqrt{2})^2 - 2 \times 2\sqrt{2} \times 1 + 1^2 - \{(\sqrt{2})^2 + 2 \times \sqrt{2} \times 3 + 3^2\} \\& = 8 - 4\sqrt{2} + 1 - (2 + 6\sqrt{2} + 9) \\& = -2 - 10\sqrt{2}\end{aligned}$$

$$\begin{aligned}(2) \quad & \left(\frac{\sqrt{2} + 1}{2}\right)^2 - \left(\frac{\sqrt{2} - 1}{2}\right)^2 = \frac{(\sqrt{2} + 1)^2}{4} - \frac{(\sqrt{2} - 1)^2}{4} \\& = \frac{(\sqrt{2})^2 + 2 \times \sqrt{2} \times 1 + 1^2}{4} - \frac{(\sqrt{2})^2 - 2 \times \sqrt{2} \times 1 + 1^2}{4} \\& = \frac{4\sqrt{2}}{4} = \sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{別解} \quad & \left(\frac{\sqrt{2} + 1}{2}\right)^2 - \left(\frac{\sqrt{2} - 1}{2}\right)^2 = \left(\frac{\sqrt{2} + 1}{2} + \frac{\sqrt{2} - 1}{2}\right) \left(\frac{\sqrt{2} + 1}{2} - \frac{\sqrt{2} - 1}{2}\right) \\& = \sqrt{2} \times 1 = \sqrt{2}\end{aligned}$$

$$(3) \quad (3\sqrt{2} + 2\sqrt{3})^2 = (3\sqrt{2})^2 + 2 \times 3\sqrt{2} \times 2\sqrt{3} + (2\sqrt{3})^2 = 18 + 12\sqrt{6} + 12 = 6(5 + 2\sqrt{6})$$

$$\begin{aligned}\text{よって} \quad & (5 - 2\sqrt{6})(3\sqrt{2} + 2\sqrt{3})^2 = (5 - 2\sqrt{6}) \times 6(5 + 2\sqrt{6}) \\& = 6\{5^2 - (2\sqrt{6})^2\} \\& = 6(25 - 24) = 6\end{aligned}$$

$$\begin{aligned}(4) \quad & (\sqrt{2} + \sqrt{3} - \sqrt{5})^2 = \{(\sqrt{2} + \sqrt{3}) - \sqrt{5}\}^2 \\& = (\sqrt{2} + \sqrt{3})^2 - 2(\sqrt{2} + \sqrt{3})\sqrt{5} + (\sqrt{5})^2 \\& = 2 + 2\sqrt{6} + 3 - 2\sqrt{10} - 2\sqrt{15} + 5 \\& = 10 + 2\sqrt{6} - 2\sqrt{10} - 2\sqrt{15}\end{aligned}$$

$$\begin{aligned}\text{別解} \quad & (\sqrt{2} + \sqrt{3} - \sqrt{5})^2 \\& = (\sqrt{2})^2 + (\sqrt{3})^2 + (-\sqrt{5})^2 + 2\sqrt{2}\sqrt{3} + 2\sqrt{3} \times (-\sqrt{5}) + 2(-\sqrt{5}) \times \sqrt{2} \\& = 2 + 3 + 5 + 2\sqrt{6} - 2\sqrt{15} - 2\sqrt{10} \\& = 10 + 2\sqrt{6} - 2\sqrt{10} - 2\sqrt{15}\end{aligned}$$

$$\begin{aligned}(5) \quad & (1 + \sqrt{2} - \sqrt{3})(1 - \sqrt{2} + \sqrt{3})(1 + \sqrt{2} + \sqrt{3})^2 \\& = \{(1 + \sqrt{2}) - \sqrt{3}\} \{(1 + \sqrt{2}) + \sqrt{3}\} \times \{(1 + \sqrt{3}) - \sqrt{2}\} \{(1 + \sqrt{3}) + \sqrt{2}\} \\& = \{(1 + \sqrt{2})^2 - (\sqrt{3})^2\} \{(1 + \sqrt{3})^2 - (\sqrt{2})^2\} \\& = (1 + 2\sqrt{2} + 2 - 3)(1 + 2\sqrt{3} + 3 - 2) \\& = 2\sqrt{2}(2 + 2\sqrt{3}) = 4\sqrt{2} + 4\sqrt{6}\end{aligned}$$

$$\begin{aligned}(6) \quad & \frac{1}{12} \{(\sqrt{3} + \sqrt{15} + \sqrt{21})^2 - (\sqrt{3} - \sqrt{15} - \sqrt{21})^2\} \\& = \frac{1}{12} \{[(\sqrt{3} + \sqrt{15} + \sqrt{21}) + (\sqrt{3} - \sqrt{15} - \sqrt{21})] \\& \quad \times [(\sqrt{3} + \sqrt{15} + \sqrt{21}) - (\sqrt{3} - \sqrt{15} - \sqrt{21})]\} \\& = \frac{1}{12} \times 2\sqrt{3} \times 2(\sqrt{15} + \sqrt{21}) \\& = \frac{1}{3} (3\sqrt{5} + 3\sqrt{7}) = \sqrt{5} + \sqrt{7}\end{aligned}$$

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解答 (1) $3 - \sqrt{5}$ (2) $\frac{\sqrt{5} + \sqrt{2}}{3}$ (3) $6 + \sqrt{35}$ (4) $4 - \sqrt{15}$
(5) $\frac{13 - 5\sqrt{5}}{11}$ (6) $\frac{7 + 2\sqrt{6}}{5}$

解説

$$\begin{aligned}(1) \quad & \frac{4}{3 + \sqrt{5}} = \frac{4(3 - \sqrt{5})}{(3 + \sqrt{5})(3 - \sqrt{5})} = \frac{4(3 - \sqrt{5})}{3^2 - (\sqrt{5})^2} \\& = \frac{4(3 - \sqrt{5})}{9 - 5} = \frac{4(3 - \sqrt{5})}{4} = 3 - \sqrt{5}\end{aligned}$$

$$\begin{aligned}(2) \quad & \frac{1}{\sqrt{5} - \sqrt{2}} = \frac{\sqrt{5} + \sqrt{2}}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})} = \frac{\sqrt{5} + \sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2} \\& = \frac{\sqrt{5} + \sqrt{2}}{5 - 2} = \frac{\sqrt{5} + \sqrt{2}}{3}\end{aligned}$$

$$\begin{aligned}(3) \quad & \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} = \frac{(\sqrt{7} + \sqrt{5})(\sqrt{7} + \sqrt{5})}{(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5})} = \frac{(\sqrt{7} + \sqrt{5})^2}{(\sqrt{7})^2 - (\sqrt{5})^2} \\& = \frac{7 + 2\sqrt{35} + 5}{7 - 5} = \frac{12 + 2\sqrt{35}}{2} \\& = \frac{2(6 + \sqrt{35})}{2} = 6 + \sqrt{35}\end{aligned}$$

$$\begin{aligned}(4) \quad & \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{(\sqrt{5} - \sqrt{3})(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})} = \frac{(\sqrt{5} - \sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2} \\& = \frac{5 - 2\sqrt{15} + 3}{5 - 3} = \frac{8 - 2\sqrt{15}}{2} \\& = \frac{2(4 - \sqrt{15})}{2} = 4 - \sqrt{15}\end{aligned}$$

$$\begin{aligned}(5) \quad & \frac{\sqrt{5} - 1}{2\sqrt{5} + 3} = \frac{(\sqrt{5} - 1)(2\sqrt{5} - 3)}{(2\sqrt{5} + 3)(2\sqrt{5} - 3)} \\& = \frac{2(\sqrt{5})^2 + (-3 - 2)\sqrt{5} + (-1) \times (-3)}{(2\sqrt{5})^2 - 3^2} \\& = \frac{10 - 5\sqrt{5} + 3}{20 - 9} = \frac{13 - 5\sqrt{5}}{11}\end{aligned}$$

$$\begin{aligned}(6) \quad & \frac{3\sqrt{2} + \sqrt{3}}{3\sqrt{2} - \sqrt{3}} = \frac{(3\sqrt{2} + \sqrt{3})(3\sqrt{2} + \sqrt{3})}{(3\sqrt{2} - \sqrt{3})(3\sqrt{2} + \sqrt{3})} = \frac{(3\sqrt{2} + \sqrt{3})^2}{(3\sqrt{2})^2 - (\sqrt{3})^2} \\& = \frac{18 + 6\sqrt{6} + 3}{18 - 3} = \frac{21 + 6\sqrt{6}}{15} \\& = \frac{3(7 + 2\sqrt{6})}{15} = \frac{7 + 2\sqrt{6}}{5}\end{aligned}$$

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【解答】 (1) $2\sqrt{6}$ (2) $\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}$

【解説】

$$\begin{aligned} (1) \quad & (\sqrt{2} + \sqrt{3} + \sqrt{5})(\sqrt{2} + \sqrt{3} - \sqrt{5}) = ((\sqrt{2} + \sqrt{3}) + \sqrt{5})((\sqrt{2} + \sqrt{3}) - \sqrt{5}) \\ & = (\sqrt{2} + \sqrt{3})^2 - (\sqrt{5})^2 \\ & = 2 + 2\sqrt{6} + 3 - 5 = 2\sqrt{6} \\ (2) \quad & \frac{1}{\sqrt{2} + \sqrt{3} + \sqrt{5}} = \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{(\sqrt{2} + \sqrt{3} + \sqrt{5})(\sqrt{2} + \sqrt{3} - \sqrt{5})} \\ & = \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{2\sqrt{6}} \\ & = \frac{(\sqrt{2} + \sqrt{3} - \sqrt{5}) \times \sqrt{6}}{2\sqrt{6} \times \sqrt{6}} \\ & = \frac{2\sqrt{3} + 3\sqrt{2} - \sqrt{30}}{12} \end{aligned}$$

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【解答】 (1) 5 (2) 24 (3) 54

【解説】

$$\begin{aligned} (1) \quad & x^2 + 4x + 6 = x(x+4) + 6 = (\sqrt{3}-2)((\sqrt{3}-2)+4) + 6 \\ & = (\sqrt{3}-2)(\sqrt{3}+2) + 6 = (\sqrt{3})^2 - 2^2 + 6 \\ & = 3 - 4 + 6 = 5 \end{aligned}$$

【別解】 1 $x^2 + 4x + 6 = (x+2)^2 + 2 = (\sqrt{3}-2+2)^2 + 2 = (\sqrt{3})^2 + 2 = 5$

【別解】 2 $x = \sqrt{3}-2$ から $x+2 = \sqrt{3}$
両辺を2乗すると $(x+2)^2 = (\sqrt{3})^2$
すなわち $x^2 + 4x + 4 = 3$
よって $x^2 + 4x = -1$
したがって $x^2 + 4x + 6 = -1 + 6 = 5$

$$\begin{aligned} (2) \quad & x^2 - 6x + 5 = (x-1)(x-5) = (2\sqrt{7}+3-1)(2\sqrt{7}+3-5) \\ & = (2\sqrt{7}+2)(2\sqrt{7}-2) = (2\sqrt{7})^2 - 2^2 \\ & = 28 - 4 = 24 \end{aligned}$$

【別解】 1 $x^2 - 6x + 5 = (x-3)^2 - 4 = (2\sqrt{7}+3-3)^2 - 4 = (2\sqrt{7})^2 - 4 = 28 - 4 = 24$

【別解】 2 $x = 2\sqrt{7}+3$ から $x-3 = 2\sqrt{7}$
両辺を2乗すると $(x-3)^2 = (2\sqrt{7})^2$
すなわち $x^2 - 6x + 9 = 28$
よって $x^2 - 6x = 19$
したがって $x^2 - 6x + 5 = 19 + 5 = 24$

$$\begin{aligned} (3) \quad & x+y = (3+\sqrt{3}) + (3-\sqrt{3}) = 6 \\ & xy = (3+\sqrt{3})(3-\sqrt{3}) = 9-3=6 \\ \text{よって} \quad & x^2 + 5xy + y^2 = (x+y)^2 - 2xy + 5xy \\ & = (x+y)^2 + 3xy \\ & = 6^2 + 3 \times 6 = 36 + 18 = 54 \end{aligned}$$

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【解答】 -6

【解説】

$$\begin{aligned} (a+b)(a-b) - \frac{(2a+b)(a-2b)}{2} &= a^2 - b^2 - \frac{2a^2 - 3ab - 2b^2}{2} \\ &= \frac{3ab}{2} \\ &= \frac{3(3\sqrt{5}+7)(3\sqrt{5}-7)}{2} \\ &= \frac{3\{(3\sqrt{5})^2 - 7^2\}}{2} = \frac{3(45-49)}{2} \\ &= \frac{3 \times (-4)}{2} = -6 \end{aligned}$$

49

【解答】 (1) 23 (2) $a=105$ (3) $x=26$

【解説】

$$\begin{aligned} (1) \quad & 11 = \sqrt{121}, \quad 12 = \sqrt{144} \text{ であるから } \sqrt{121} \leq \sqrt{a} < \sqrt{144} \\ \text{よって} \quad & 121 \leq a < 144 \\ a \text{ は自然数であるから} \quad & 121 \leq a \leq 143 \\ \text{よって, 求める自然数 } a \text{ の個数は} \quad & 143 - 121 + 1 = 23 \\ (2) \quad & \sqrt{2^3 \times 3^4 \times 5 \times 6^3 \times 7^3 \times a} \text{ が自然数となるのは, } 2^3 \times 3^4 \times 5 \times 6^3 \times 7^3 \times a \text{ が自然数の} \\ & \text{2乗になるときである。} \\ \text{自然数の2乗になるためには, } \quad & 2^3 \times 3^4 \times 5 \times 6^3 \times 7^3 \times a \text{ を素因数分解したときに, すべて} \\ & \text{の指数が偶数になっている必要がある。} \\ 2^3 \times 3^4 \times 5 \times 6^3 \times 7^3 \times a = 2^5 \times 3^7 \times 5 \times 7^3 \times a \text{ であるから, このような条件を満たす } a \text{ で} \\ \text{最も小さいものは} \quad & a = 3 \times 5 \times 7 = 105 \end{aligned}$$

$$(3) \quad \sqrt{\frac{936}{x}} \text{ が自然数となるのは, } \frac{936}{x} \text{ が自然数の2乗になるときである。}$$

$$\begin{aligned} \text{自然数の2乗になるためには, } \quad & \frac{936}{x} \text{ を素因数分解したときに, すべての指数が偶数に} \\ \text{なっている必要がある。} \\ \frac{936}{x} = \frac{2^3 \times 3^2 \times 13}{x} \text{ であるから, このような条件を満たす } x \text{ で最も小さいものは} \\ & x = 2 \times 13 = 26 \end{aligned}$$

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【解答】 (1) $19+2\sqrt{3}$ (2) 4 (3) $\frac{4}{7}$

【解説】

$$\begin{aligned} (1) \quad & 2\sqrt{3} = \sqrt{12}, \quad \sqrt{9} < \sqrt{12} < \sqrt{16} \text{ であるから } 3 < 2\sqrt{3} < 4 \\ \text{各辺に } -1 \text{ をかけて} \quad & -4 < -2\sqrt{3} < -3 \\ \text{各辺に } 7 \text{ をたして} \quad & 3 < 7-2\sqrt{3} < 4 \\ \text{よって} \quad & a=3, \quad b=7-2\sqrt{3}-3=4-2\sqrt{3} \\ \text{したがって} \quad & 3a^2 - 3ab + b^2 = 3 \times 3^2 - 3 \times 3(4-2\sqrt{3}) + (4-2\sqrt{3})^2 \\ & = 27 - (36 - 18\sqrt{3}) + (16 - 16\sqrt{3} + 12) \\ & = 19 + 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} (2) \quad & \sqrt{1} < \sqrt{3} < \sqrt{4} \text{ であるから } 1 < \sqrt{3} < 2 \\ \text{各辺に } -1 \text{ をかけて} \quad & -2 < -\sqrt{3} < -1 \\ \text{各辺に } 4 \text{ をたして} \quad & 2 < 4-\sqrt{3} < 3 \\ \text{よって} \quad & a=2, \quad b=4-\sqrt{3}-2=2-\sqrt{3} \\ \text{したがって} \quad & \frac{1}{b} + \frac{1}{2a-b} = \frac{1}{2-\sqrt{3}} + \frac{1}{2 \times 2 - (2-\sqrt{3})} \\ & = \frac{1}{2-\sqrt{3}} + \frac{1}{2+\sqrt{3}} \quad \dots\dots (*) \\ & = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})} + \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} \\ & = \frac{2+\sqrt{3}}{2^2-(\sqrt{3})^2} + \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} \\ & = (2+\sqrt{3}) + (2-\sqrt{3}) = 4 \end{aligned}$$

【別解】 $(*) = \frac{(2+\sqrt{3})+(2-\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{4}{2^2-(\sqrt{3})^2} = 4$

$$\begin{aligned} (3) \quad & \frac{7}{3-\sqrt{2}} = \frac{7(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{7(3+\sqrt{2})}{3^2-(\sqrt{2})^2} \\ & = \frac{7(3+\sqrt{2})}{9-2} = 3 + \sqrt{2} \end{aligned}$$

$$\begin{aligned} \sqrt{1} < \sqrt{2} < \sqrt{4} \text{ であるから } 1 < \sqrt{2} < 2 \\ \text{各辺に } 3 \text{ をたして} \quad & 4 < 3+\sqrt{2} < 5 \\ \text{よって} \quad & a=4, \quad b=3+\sqrt{2}-4=\sqrt{2}-1 \\ \text{したがって} \quad & \frac{1}{a+b+1} + \frac{1}{a-b-1} = \frac{1}{4+(\sqrt{2}-1)+1} + \frac{1}{4-(\sqrt{2}-1)-1} \\ & = \frac{1}{4+\sqrt{2}} + \frac{1}{4-\sqrt{2}} \quad \dots\dots (*) \\ & = \frac{4-\sqrt{2}}{(4+\sqrt{2})(4-\sqrt{2})} + \frac{4+\sqrt{2}}{(4-\sqrt{2})(4+\sqrt{2})} \\ & = \frac{4-\sqrt{2}}{4^2-(\sqrt{2})^2} + \frac{4+\sqrt{2}}{4^2-(\sqrt{2})^2} \\ & = \frac{4-\sqrt{2}}{16-2} + \frac{4+\sqrt{2}}{16-2} \\ & = \frac{4-\sqrt{2}}{14} + \frac{4+\sqrt{2}}{14} \\ & = \frac{8}{14} = \frac{4}{7} \end{aligned}$$

【別解】 $(*) = \frac{(4-\sqrt{2})+(4+\sqrt{2})}{(4+\sqrt{2})(4-\sqrt{2})} = \frac{8}{4^2-(\sqrt{2})^2} = \frac{8}{14} = \frac{4}{7}$

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【解答】 (1) $0.\dot{4}\dot{5}$ (2) $0.\dot{7}\dot{0}\dot{3}$ (3) $0.\dot{3}0\dot{6}$ (4) $0.4285\dot{7}\dot{1}$

【解説】

$$\begin{aligned} (1) \quad & \frac{5}{11} = 0.454545\dots\dots = 0.\dot{4}\dot{5} \\ (2) \quad & \frac{19}{27} = 0.703703703\dots\dots = 0.\dot{7}\dot{0}\dot{3} \\ (3) \quad & \frac{34}{111} = 0.306306306\dots\dots = 0.\dot{3}0\dot{6} \end{aligned}$$

(4) $\frac{3}{7} = 0.4285714285714\cdots = 0.\dot{4}2857\dot{1}$

52

【解答】 (1) $\frac{7}{9}$ (2) $\frac{41}{333}$ (3) $\frac{8}{55}$ (4) $\frac{5231}{1665}$

【解説】

(1) $0.\dot{7} = x$ とおくと

$$\begin{array}{r} 10x = 7.777\cdots \\ -) \quad x = 0.777\cdots \\ \hline 9x = 7 \end{array}$$

$x = \frac{7}{9}$

(2) $0.\dot{1}2\dot{3} = x$ とおくと

$$\begin{array}{r} 1000x = 123.123123\cdots \\ -) \quad x = 0.123123\cdots \\ \hline 999x = 123 \end{array}$$

$x = \frac{123}{999} = \frac{41}{333}$

(3) $0.1\dot{4}\dot{5} = x$ とおくと

$$\begin{array}{r} 100x = 14.54545\cdots \\ -) \quad x = 0.14545\cdots \\ \hline 99x = 14.4 \end{array}$$

$x = \frac{14.4}{99} = \frac{144}{990} = \frac{8}{55}$

(4) $3.1\dot{4}1\dot{7} = x$ とおくと

$$\begin{array}{r} 1000x = 3141.7417417\cdots \\ -) \quad x = 3.1417417\cdots \\ \hline 999x = 3138.6 \end{array}$$

$x = \frac{3138.6}{999} = \frac{31386}{9990} = \frac{5231}{1665}$